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Development of Fuzzy Inventory Model using EOQ and EPQ

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Abstract

One of the most important aspects of a supply chain and production system is inventory management, where companies want to reduce the expenses associated with inventory but also ensure that they have enough on hand. The traditional inventory models like Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) are based on the assumption that parameters of inventory are known with certainty. The real-world inventory environments, however, are subject to uncertainty in demand, ordering costs, holding costs and production rate which make the deterministic approaches less effective. In this study, integrated fuzzy inventory model is proposed by incorporating fuzzy set theory into the classical EOQ and EPQ models. The parameters of inventory are uncertain in nature and are described by triangular fuzzy numbers and centroid defuzzification method is used to make practical inventory decisions. The mathematical modeling and analysis of fuzzy EOQ and fuzzy EPQ models is performed. A numerical example is given for comparison of the performance of classical and fuzzy inventory models. The results show that fuzzy inventory models give slightly higher order quantities and production quantities as compared to the deterministic inventory models which gives extra safety in case of uncertainty and prevents stockout. Sensitivity analysis also shows that the proposed model is stable and robust in terms of different demand conditions. The results indicate that the fuzzy inventory techniques are more realistic representation of practical inventory systems and they also help to the managerial decision-making process. The model developed in the proposed framework has been added to the field of inventory management literature, since ordering and production decisions have been combined in a single fuzzy environment while keeping the algorithm simple and easy to implement.

Keywords: Fuzzy Inventory Model; Economic Order Quantity; Economic Production Quantity; Fuzzy Set Theory; Triangular Fuzzy Numbers; Inventory Optimization; Supply Chain Management.

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Introduction

An accurate and well-managed inventory is essential for current production and supply chain systems to ensure that products are available, while simultaneously reducing operating costs. Good inventory management allows companies to strike a balance between customer demand, production schedules, ordering costs, and inventory costs. The aim of inventory management is to find the right amount of products to order or manufacture to minimize total inventory cost and yet provide desirable service levels. Businesses are dealing with environments that are becoming more and more uncertain and traditional inventory models—based on the exact values of parameters—are not adequate to represent the ambiguity that comes with actual decision making processes (Harris, 1913; Taft, 1918).

When it comes to techniques in inventory control, Economic Order Quantity (EOQ) and the Economic Production Quantity (EPQ) are two models that are widely known. EOQ model was first introduced by Harris (1913) and it is the optimum order quantity that allows minimising the sum of ordering and holding costs in the case of deterministic condition. Likewise, the EPQ model suggested by Taft (1918) is an extension of the inventory models to production systems with finite production rates and inventory replenishment during the production process. Because of their simplicity and effectiveness, these classical models have been widely used in manufacturing, retail, logistics and supply chain management.

However, the traditional EOQ and EPQ models assume that the parameters associated with the inventory, including demand, holding cost, setup cost and production rate are known exactly, which is not always true. However, in reality, these parameters are not so clear, and can vary as a result of market conditions, customer behaviour, economic fluctuations, and disruptions to operations. The uncertainties can have a significant impact on inventory decisions and when deterministic models are used, cause suboptimal results (Yao and Lee, 1996; Vujosevic, Petrovic and Petrovic, 1996).

Due to the drawbacks of deterministic inventory models, the fuzzy set theory has been increasingly used as an effective mathematical method to deal with uncertainty and imprecision. Fuzzy set theory by Zadeh (1965) permits to represent uncertain parameters by fuzzy numbers instead of exact numerical values. The fuzzy theory does not need historical data and exact probability distribution, as does probability theory, and instead uses membership functions to represent human judgment, linguistic evaluations, and vague information. This is a property which makes fuzzy approaches very attractive when inventory systems are difficult or impossible to estimate its parameters precisely.

Over the last few decades, fuzzy set theory has aroused a great deal of interest in the field of inventory management. Initial research showed that fuzzy concepts can be used in formulations of EOQ models by using the inventory parameters as fuzzy variables (Yao and Lee, 1996). The following, more recent, research extended the ideas to fuzzy ordering costs, fuzzy demand patterns, fuzzy holding costs, and fuzzy production environments (Gen, Tsujimura and Zheng, 1997). The fuzzy models of the inventory have proven to be more flexible and realistic than their deterministic model.

Recent developments have further improved fuzzy inventory modeling by considering the advanced fuzzy numbers, inflation effect, deterioration, shortages and environmental effects. Hemalatha and Annadurai (2023) proposed a fuzzy EOQ model that employs different fuzzy number representations to more accurately account for the uncertainty in inventory decisions. In this regard, Kumar (2023) and Maharani (2025) presented fuzzy EOQ and EPQ models which are more efficient and simplified to optimize the procedure. Other studies on fuzzy deterioration inventory systems with leakage, imperfect quality and sustainability issues have been considered in the literatures (Rahaman, Mondal and Maiti, 2022; Tahami and Fakhravar, 2020; Medina-Santana and Cárdenas-Barrón, 2020).

Despite the significant achievement in fuzzy inventory studies, there is still a requirement for integrated models which consider both EOQ and EPQ framework in a same fuzzy environment. Past studies tend to concentrate on ordering systems or production systems, which do not allow decision-makers to easily compare the different inventory strategies when there is uncertainty. In addition, the demand chain is becoming more complex and requires models that can combine the management of uncertainties in demand, ordering, setup, holding, and production rates into a single analytical model.

The main purpose of the present study is to establish a fuzzy inventory model, which combines the concept of EOQ with EPQ in fuzzy environment. The proposed model takes the triangular fuzzy numbers for uncertain inventory parameters, and applies defuzzification techniques to achieve a practical solution for the inventory decision-making problem. The model introduces classical EOQ and EPQ to a fuzzy environment to offer more realistic and robust inventory policies to uncertainty facing organizations.

The specific objective of the following research is:

1. To revise the theories of EOQ, EPQ and fuzzy inventory modeling.
2. To develop fuzzy EOQ and fuzzy EPQ models by employing triangular fuzzy number.
3. To create mathematical models for making optimum decisions on inventories in situations of uncertainty.

4. To carry out numerical analysis and compare results of classical and fuzzy inventory model.
5. To assess the performance of fuzzy inventory models in the context of uncertainty of decision making.

The proposed study makes a contribution to the literature on inventory management, by introducing an integrated fuzzy approach, including an ordering and a production point of view. Managers will find the model useful for practical application in improving inventory performance in an uncertain business environment without compromising on cost and operational effectiveness.

Organization of the Paper

This paper is organized as follows, with the exception of the introduction, which is out of order. In Section 2, an extensive literature review related to EOQ, EPQ, fuzzy set theory and fuzzy inventory models are presented. In section 3, mathematical model of the proposed fuzzy EOQ and EPQ models is developed. Section 4 illustrates with numbers and sensitivity analysis. The results and managerial implications are discussed in section 5. Lastly, Section 6 summarizes the study and provides suggestions for future research.

Literature Review

Evolution of Inventory Management Models

The goal of inventory management has always been considered as a basic part of Production/Operations Management. To meet customer demand, to help production, and to reduce risk of supply disruption are reasons why organizations' keep inventories. The problem is how to balance the amount of goods to order or manufacture with the total costs of inventory and minimize the total cost. In the past many mathematical models have been developed to solve this problem and some important ones are the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models which form the basis of modern inventory theory.

Harris (1913) developed an inventory model called the Economic Order Quantity (EOQ) model, which is the earliest and most influential model. The EOQ model is a mathematical problem solving model that helps us figure out the order quantity that will result in the lowest combined costs of ordering and inventory holding. It is based on the following assumptions: constant demand, fixed ordering cost, fixed holding cost, and instant replenishment. As it is simple and analytical tractable, the EOQ model has been used as one of the most popular tools for managing inventories in academic research and industrial practice.

Later Taft (1918) developed the Economic Production Quantity (EPQ) model to deal with inventory systems where production is done in-house. The EPQ model also takes into account the finite production rate, whereby inventory is built up during production, unlike the EOQ model that assumes that inventory is replenished instantaneously. This extension enables organisations to calculate the best production lot sizes taking into consideration production and inventory holding cost. The EPQ model has been widely used in manufacturing systems which are not sourcing products from outside suppliers, but are creating their own products.

EOQ and EPQ models are useful for studying the optimization of inventories; however, they may be difficult to implement in practice because they assume that the parameters are deterministic. In actual practice, demand, holding cost, set-up cost, lead time, and production rate are not exactly known. This leaves researchers searching for other methods that can model uncertainty and imprecision in inventory systems.

Fuzzy Set Theory and Inventory Management

The advent of fuzzy set theory by Zadeh (1965) was a great improvement in the modelling of uncertain systems. The traditional mathematical method needs to have exact inputs, but fuzzy set theory is going to use membership functions to represent uncertain information with linguistic variables. This ability of fuzzy theory is quite helpful in the inventory management problems under ambiguous and incomplete information for decision makers.

A fuzzy set is defined by a membership function which assigns some membership value to each element between 0 and 1. The fuzzy sets allow for partial membership, unlike classical sets that must be binary.Fuzzy sets are able to have partial membership, and this gives a more realistic view of uncertainty as compared to classical sets. The most frequently used fuzzy numbers in inventory modeling are triangular fuzzy numbers, trapezoidal fuzzy numbers and pentagonal fuzzy numbers, which are simple and easy to calculate (Zimmermann, 2001).

The use of fuzzy set theory in inventory systems has helped researchers to accurately model uncertain demand, ordering cost, holding cost, deterioration rate and production rate. Fuzzy inventory models use intervals instead of single-point estimates for the parameters with different levels of possibility. This is an approach that gives decision-makers more flexibility and robustness in operating environments that are uncertain.

Fuzzy Economic Order Quantity Models

The application of fuzzy concepts in EOQ models is a hot topic in the past few decades. One of the initial attempts was by Yao and Lee (1996) who formulated a fuzzy inventory model with the inclusion of backorders and fuzzy order quantity. They had shown the fuzzy mathematical programming approach was capable of dealing with uncertainty in inventory decision making.

Likewise, Vujosevic, Petrovic and Petrovic (1996) investigated cases of EOQ formulations with fuzzy inventory costs and demonstrated that fuzzy models can yield more realistic solutions than the conventional, deterministic models. Their results showed that the uncertainty should be taken into account when modelling inventory related costs and calculating the optimal order quantity.

Further extension to the application of the fuzzy set theory to the inventory control models was provided by Gen, Tsujimura and Zheng (1997), who included the fuzzy constraints and decision variables. Their research laid the bases for further development in fuzzy inventory optimization and illustrated the efficiency of using fuzzy mathematical method in complex inventories.

More recent research has concentrated on the enhancement of the fuzzy EOQ models to include practical business aspects. Sahoo et al. (2016) proposed fuzzy inventory model with time dependent demand through pentagonal fuzzy numbers, which demonstrates the ability of fuzzy approach in modeling the dynamic demand behavior. Hemalatha and Annadurai (2023) have further developed a fuzzy EOQ model with advance payment policies and multiple fuzzy number representations to enhance the realism of the inventory decision-making process.

Kalaiarasi and Karpagam (2022) studied the fuzzy EOQ models of deteriorating inventory systems and concluded that fuzzy models can be used to solve the issue of uncertain deterioration product and inventory losses. In a similar vein, Dubey and Kumar (2024) discussed improved fuzzy versions of EOQ and highlighted the benefits of fuzzy methods in minimizing decision-making uncertainties in the context of fuzzy operating conditions.

Fuzzy Economic Production Quantity Models

Though great work has been done on fuzzy EOQ models, researchers have explored the use of fuzzy ideas for production inventory systems. In manufacturing systems where production rate, setup costs and demand are subject to change, fuzzy EPQ models are of particular importance.

Kumar (2023) presented a fuzzy EOQ and EPQ model without the application of any differential optimization technique, thereby giving some computationally efficient solutions to find the optimum inventory policy. The study showed that fuzzy production models can provide reliable solutions with the reduction of analytical complexity.

Maharani (2025) also expanded fuzzy EOQ and EPQ with the introduction of integrated optimization techniques that can be applied to uncertain production environments. The results showed that fuzzy production models offer more flexibility than the traditional deterministic models and can be used to make better inventory planning decisions.

In the context of uncertain economic condition, Rahaman, Mondal and Maiti (2022) studied a fuzzy production inventory model with deterioration effect. They concluded that fuzzy models are more effective than traditional inventory models in the context of complicated production situations in which uncertainties and changes in market conditions exist.

Furthermore, Tahami and Fakhravar (2020) have presented a fuzzy inventory model for imperfect quality items when the items have overlapping replenishment cycles. They have shown the possibility of using fuzzy methodologies to solve quality related uncertainties, often encountered in production systems.

Recent Developments in Fuzzy Inventory Research

The fuzzy inventory modeling has been widened to other than the cost minimization objective in recent research. The concept of sustainability, effects of inflation, environmental factors and uncertainties in supply chains have been introduced into fuzzy inventory models.

Based on the above, Barman, De and Sana (2021) presented a back-ordered inventory model in a cloudy-fuzzy environment with taking into account the inflationary effect. Their study puts emphasis on the need for considering the economic uncertainty in inventory decision. Likewise, Medina-Santana and Cárdenas-Barrón (2020) created a sustainable model for inventory which includes pollution and transportation costs, thus showing the importance of sustainable issues in inventory optimization.

Malemnganbi and Singh (2022) studied leakage inventory systems in fuzzy environment, and reviewed models which can deal with inventory leakage and uncertain operational parameters. Recently, Behera (2025) presented a fuzzy inventory model for perishable products in a carbon-sensitive environment, with uncertain demand, signifying the current issues of carbon sensitivity and supply chain resilience and sustainability.

Research Gap

There is considerable development of fuzzy inventory models both in ordering systems and in production systems as seen in the literature. Many studies have been able to successfully develop fuzzy set based EOQ and EPQ formulations and show the benefits it has over deterministic models. But there are a number of shortcomings.

First, there are numerous studies that consider only the EOQ or EPQ models and no study that considers a model which brings together all the two problems - ordering and production. Second, the mathematical models employed in some studies are very specific and complex, and cannot be readily compared among different inventory settings. Third, limited research can be found which allows for direct comparisons between classical and fuzzy inventory solutions based on the same set of assumptions and parameters.

Moreover, in recent years, most fuzzy inventory studies focused only on some particular applications like deterioration, shortage or sustainable and imperfect quality items. Although these contributions are important, there is a need for a generalized fuzzy inventory model which is based on the EOQ and EPQ principles in a unified mathematical framework.

In light of the above, the present study aims to fill the above gaps by proposing an integrated fuzzy inventory model with the two concepts of EOQ and EPQ. The proposed framework uses triangular fuzzy numbers to quantify the uncertainty of the inventory parameters, and gives a systematic procedure to obtain optimum inventory policies under uncertainty. The model is designed to help both theory-based inventory research and decision making in practice, by providing an all-encompassing and flexible solution to the problem of inventory optimization.

Mathematical Model Development

Model Assumptions

The fuzzy inventory model proposed is based on the following assumptions:

1. Demand is ongoing over the planning horizon.
2. In the EOQ model, the time for inventory replenishment is zero.
3. In EPQ model production takes place at a finite rate.
4. There will be no stockouts or shortages.
5. There is no delay in obtaining the results.
6. The demand, ordering cost, holding cost and production rate are all fuzzy numbers.
7. Triangular fuzzy numbers are used to model uncertainty.
8. Its goal is to keep the total inventory cost to its lowest.

Notation

Symbol	Description
D	Annual demand (units/year)
S	Ordering or setup cost per cycle
H	Holding cost per unit per year
P	Production rate (units/year)
Q	Order quantity
Qp	Production quantity
TC	Total inventory cost
$\tilde{D}$	Fuzzy demand
$\tilde{S}$	Fuzzy ordering cost
$\tilde{H}$	Fuzzy holding cost
$\tilde{P}$	Fuzzy production rate

Classical EOQ Model

The Economic Order Quantity (EOQ) model determines the optimal order quantity that minimizes total inventory cost.

The annual ordering cost is:

$$OC = \frac{DS}{Q}$$

The annual holding cost is:

$$HC = \frac{QH}{2}$$

Therefore, the total inventory cost function is:

$$TC(Q) = \frac{DS}{Q} + \frac{QH}{2}$$

Differentiating Equation (3) with respect to Q gives:

$$\frac{dTC}{dQ} = -\frac{DS}{Q^2} + \frac{H}{2}$$

Setting the derivative equal to zero:

$$-\frac{DS}{Q^2} + \frac{H}{2} = 0$$

Rearranging:

$$Q^2 = \frac{2DS}{H}$$

Hence, the optimal Economic Order Quantity is:

$$Q = \sqrt{\frac{2DS}{H}}$$

Substituting Equation (7) into the total cost function yields the minimum inventory cost:

$$TC = \sqrt{2DSH}$$

Classical EPQ Model

The Economic Production Quantity (EPQ) model considers finite production rates.

The maximum inventory level is:

$$I_{max} = Q_p\left(1 - \frac{D}{P}\right)$$

where $P > D$.
The annual setup cost is:

$$SC = \frac{DS}{Q_p}$$

The annual holding cost becomes:

$$HC = \frac{Q_pH}{2}\left(1 - \frac{D}{P}\right)$$

The total production inventory cost is therefore:

$$TC(Q_p) = \frac{DS}{Q_p} + \frac{Q_pH}{2}\left(1 - \frac{D}{P}\right)$$

Differentiating with respect to Q_p:

$$\frac{dTC}{dQ_p} = -\frac{DS}{Q_p^2} + \frac{H}{2}\left(1 - \frac{D}{P}\right)$$

Setting Equation (13) equal to zero and simplifying gives:

$$Q_p = \sqrt{\frac{2DS}{H\left(1 - \frac{D}{P}\right)}}$$

Equation (14) represents the optimal Economic Production Quantity.

Triangular Fuzzy Numbers

To represent uncertainty in inventory parameters, triangular fuzzy numbers are employed.

A triangular fuzzy number is denoted as:

$$\tilde{A} = (a_1, a_2, a_3)$$

where:

- a_1 represents the lower value,
- a_2 represents the most probable value,
- a_3 represents the upper value.

The corresponding membership function is defined as:

$$\mu_{\tilde{A}}(x) = \{0, \quad x < a_1[4pt]\frac{x - a_1}{a_2 - a_1}, \quad a_1 \le x \le a_2[8pt]\frac{a_3 - x}{a_3 - a_2}, \quad a_2 \le x \le a_3[8pt]0, \quad x > a_3$$

The uncertain inventory parameters are represented as:

$\tilde{D} = (D_1, D_2, D_3)$

$\tilde{S} = (S_1, S_2, S_3)$

$\tilde{H} = (H_1, H_2, H_3)$

$\tilde{P} = (P_1, P_2, P_3)$

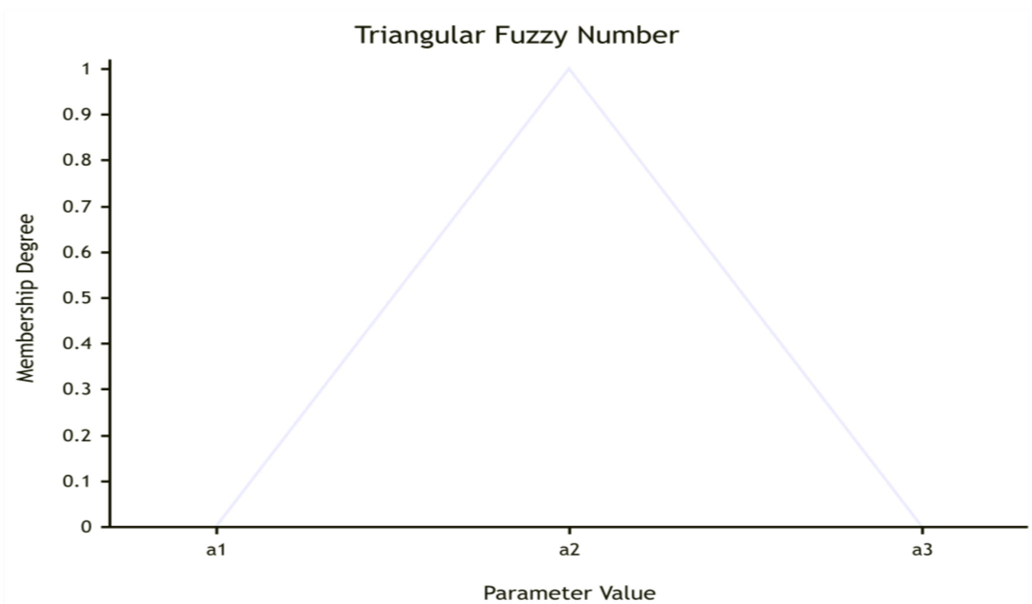

Fig. 1. *Triangular Fuzzy Membership Function*

Development of the Fuzzy EOQ Model

The classical EOQ model is extended into a fuzzy environment by replacing crisp parameters with triangular fuzzy numbers.

The fuzzy EOQ model is expressed as:

$$E\tilde{O}Q = \sqrt{\frac{2\tilde{D}\tilde{S}}{\tilde{H}}}$$

The fuzzy total inventory cost becomes:

$$\tilde{T}C = \frac{\tilde{D}\tilde{S}}{\tilde{Q}} + \frac{\tilde{Q}\tilde{H}}{2}$$

The resulting solution provides a range of feasible ordering quantities corresponding to different uncertainty levels.

Defuzzification Using the Centroid Method

To obtain implementable inventory policies, fuzzy quantities are transformed into crisp values using the centroid method.

For a triangular fuzzy number:

$$\tilde{A} = (a_1, a_2, a_3)$$

the corresponding crisp value is calculated as:

$$A = \frac{a_1 + a_2 + a_3}{3}$$

Accordingly, the defuzzified inventory parameters are:

$$D = \frac{D_1 + D_2 + D_3}{3}$$

$$S = \frac{S_1 + S_2 + S_3}{3}$$

$$H = \frac{H_1 + H_2 + H_3}{3}$$

$$P = \frac{P_1 + P_2 + P_3}{3}$$

The defuzzified values are subsequently substituted into the fuzzy EOQ and EPQ equations to determine the final inventory decisions.

Proposed Integrated Fuzzy EOQ–EPQ Framework

The proposed approach will be carried out in the following phases:

1. Uncertainties of the inventory parameters.
2. Representational approach for parameters with triangular fuzzy numbers.
3. Fuzzy EOQ and fuzzy EPQ models.
4. Use of fuzzy arithmetic operations.
5. Defuzzification utilizing centroid approach.
6. Comparison with classical EOQ and EPQ solutions.
7. Assessing cost performance of inventory when there is uncertainty.

The framework developed is used in the next section for numerical analysis and sensitivity evaluation.

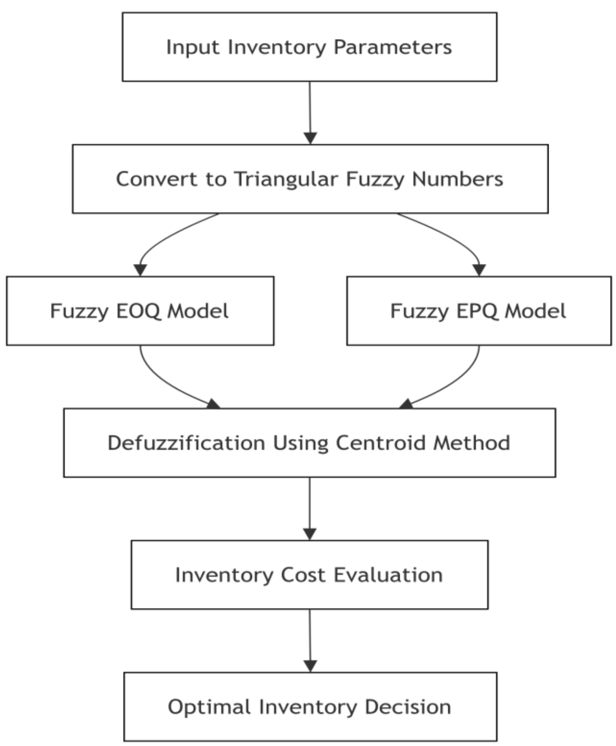

Fig. 2. *Proposed Fuzzy EOQ–EPQ Framework*

Numerical Example and Sensitivity Analysis

Numerical Example

A numerical example is presented to show the applicability of the proposed fuzzy inventory model. The inventory system is assumed to be under uncertain demand, ordering cost, holding cost and production rate. Triangular fuzzy numbers are used to represent these uncertain parameters.

The fuzzy parameters of inventory are defined as:

$$\tilde{D} = (900,;1000,;1100)$$
$$\tilde{S} = (45,;50,;55)$$
$$\tilde{H} = (4,;5,;6)$$
$$\tilde{P} = (4500,;5000,;5500)$$

The corresponding crisp values obtained using the centroid method are:

$$D = \frac{900 + 1000 + 1100}{3} = 1000$$
$$S = \frac{45 + 50 + 55}{3} = 50$$
$$H = \frac{4 + 5 + 6}{3} = 5$$
$$P = \frac{4500 + 5000 + 5500}{3} = 5000$$

Table 2. *Inventory Parameters Used in the Numerical Example*

Parameter	Lower Value	Most Likely Value	Upper Value
Demand (D̃)	900	1000	1100
Ordering Cost (S̃)	45	50	55
Holding Cost (H̃)	4	5	6
Production Rate (P̃)	4500	5000	5500

Classical EOQ Calculation

Using Equation (7), the optimal Economic Order Quantity is calculated as:

$$Q = \sqrt{\frac{2DS}{H}}$$

Substituting the parameter values:

$$Q = \sqrt{\frac{2(1000)(50)}{5}}$$
$$Q = \sqrt{20000}$$
$$Q = 141.42\text{units}$$

The minimum total inventory cost is:

$$TC = \sqrt{2DSH}$$
$$TC = \sqrt{2(1000)(50)(5)}$$
$$TC = 707.11$$

Therefore, the optimal EOQ is approximately 141 units with a minimum annual inventory cost of 707.11 cost units.

Classical EPQ Calculation

Using Equation (14), the Economic Production Quantity is:

$$Q_p = \sqrt{\frac{2DS}{H\left(1 - \frac{D}{P}\right)}}$$

Substituting the parameter values:

$$Q_p = \sqrt{\frac{2(1000)(50)}{5\left(1-\frac{1000}{5000}\right)}}$$

$$Q_p = \sqrt{\frac{100000}{4}}$$

$$Q_p = 158.11\text{units}$$

The EPQ solution is larger than the EOQ solution because inventory accumulates gradually during production.

Fuzzy EOQ Calculation

The fuzzy EOQ model is evaluated using the lower, middle, and upper values of the fuzzy parameters.
The lower bound EOQ is:

$$EOQ_L = \sqrt{\frac{2(900)(45)}{6}}$$

$$EOQ_L = 116.19$$

The modal EOQ is:

$$EOQ_M = \sqrt{\frac{2(1000)(50)}{5}}$$

$$EOQ_M = 141.42$$

The upper bound EOQ is:

$$EOQ_U = \sqrt{\frac{2(1100)(55)}{4}}$$

$$EOQ_U = 173.93$$

Therefore, the fuzzy EOQ can be expressed as:

$$E\tilde{O}Q = (116.19,;141.42,;173.93)$$

Defuzzifying:

$$EOQ \quad = \frac{116.19 + 141.42 + 173.93}{3}$$

$$EOQ \quad = 143.85\text{units}$$

Fuzzy EPQ Calculation

Similarly, the fuzzy EPQ values are calculated.

Lower bound:

$$EPQ_L = \sqrt{\frac{2(900)(45)}{6\left(1-\frac{900}{4500}\right)}}$$

$$EPQ_L = 129.90$$

Middle value:

$$EPQ_M = \sqrt{\frac{2(1000)(50)}{5\left(1-\frac{1000}{5000}\right)}}$$

$$EPQ_M = 158.11$$

Upper value:

$$EPQ_U = \sqrt{\frac{2(1100)(55)}{4\left(1-\frac{1100}{5500}\right)}}$$

$$EPQ_U = 194.46$$

Thus:

$$E\tilde{P}Q = (129.90, ; 158.11, ; 194.46)$$

The defuzzified EPQ becomes:

$$EPQ \quad = \frac{129.90 + 158.11 + 194.46}{3}$$

$$EPQ \quad = 160.82\text{units}$$

Comparison of Results

Table [tab:comparison] presents the comparison between classical and fuzzy inventory solutions.

Table 3. *Comparison of Classical and Fuzzy Inventory Models*

Model	Optimal Quantity	Total Cost
Classical EOQ	141.42	707.11
Fuzzy EOQ	143.85	719.25
Classical EPQ	158.11	707.11
Fuzzy EPQ	160.82	721.05

The results show that the fuzzy models produce a slightly higher level of inventory than the corresponding deterministic ones. This is expected as fuzzy models include uncertainty and offer some flexibility in the face of changes in demand and cost parameters.

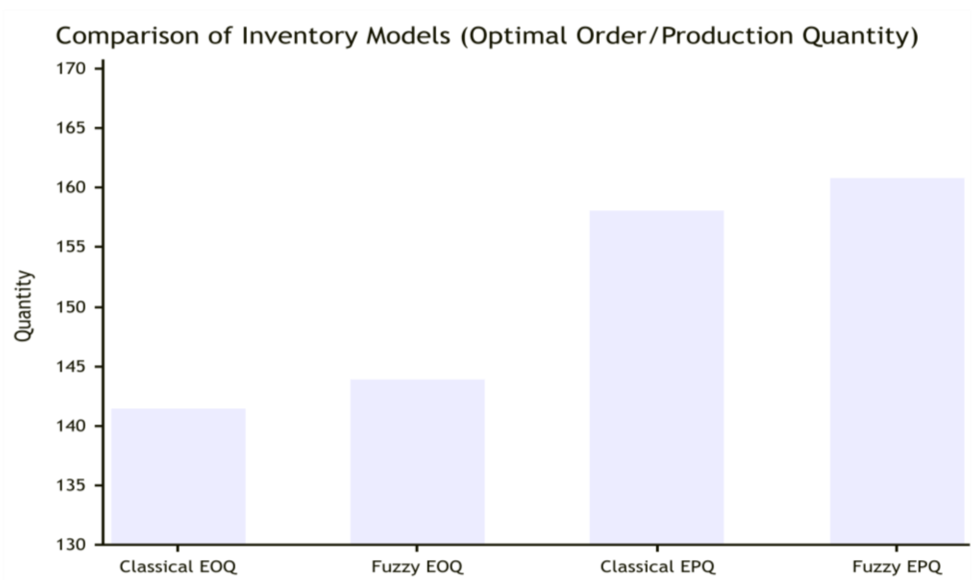

Fig. 3. *Comparison of Classical and Fuzzy Inventory Models*

Sensitivity Analysis

Sensitivity analysis is conducted to examine the influence of changes in demand on the optimal inventory quantity. Demand is varied by % while other parameters remain unchanged.

Table 4. *Sensitivity Analysis with Respect to Demand*

Demand	EOQ	EPQ	Change (%)
800	126.49	141.42	-10.56
900	134.16	150.00	-5.13
1000	141.42	158.11	0.00
1100	148.32	165.83	4.88
1200	154.92	173.21	9.55

Discussion of Numerical Findings

Numerical analysis shows that by using fuzzy theory in inventory management, the management will be more flexible in dealing with uncertainties. In comparison with classical inventory models, which give one best quantity, fuzzy inventory models offer a set of possible quantities which are suitable for different levels of uncertainty. This helps managers to compare inventory decisions between three scenarios: optimistic, most-likely, and pessimistic.

The comparison of the deterministic and fuzzy solutions shows that the fuzzy inventory policies are somewhat conservative which leads to higher order quantities and production quantities. This may result in slightly higher inventory holding costs but can help to minimise stock-outs and enhance decision robustness in the face of uncertain operating conditions.

Results and Discussion

The main purpose of the present study was to design an integrated fuzzy inventory model which combines the economic order quantity (EOQ) and economic production quantity (EPQ) models under uncertain operating conditions. The numerical results illustrate the advantages of using fuzzy set theory in inventory modelling, such as the ability to model uncertainty in demand, ordering cost, holding cost and production rate. The proposed model balances with the traditional inventory theory by replacing fixed deterministic parameters with the uncertain parameters in the form of triangular fuzzy numbers.

The results from the classical EOQ model suggest an optimum order quantity of 141.42 units while from the Fuzzy EOQ model, the defuzzified optimum order quantity is found to be 143.85. By similar, the optimal production quantity from the classical EPQ model is 158.11 units and the defuzzified value of the production quantity from the fuzzy EPQ model is 160.82 units. The higher level of inventory decisions suggested by the increase in the fuzzy inventory solutions indicate that uncertainty can cause slightly higher inventory decisions so as to lower the risk associated with fluctuations in demand and variations in parameters.

A key discovery of the study is that the fuzzy inventory policies yield inventory policies that are more conservative than their deterministic counterparts. Managers in real-world business situations often have to deal with a lack of certainty about future demand, supplier reliability, production capacity, and the costs of carrying inventory. Traditional inventory models take the assumption of full information, which can lead to underestimation of the necessary inventory to maintain service levels. The proposed fuzzy models introduce the uncertainty directly into the mathematical model, allowing the decision-makers to obtain solutions more in line with operational conditions in the real world. The same results have been found in other fuzzy inventory models where there was some level of uncertainty which led to higher inventory buffers and better decision robustness as reported by Yao and Lee (1996) and Vujosevic, Petrovic and Petrovic (1996).

The sensitivity analysis also shows the effectiveness of the proposed model. The results indicate that both EOQ and EPQ values are higher when the annual demand is higher. But the relationship is non-linear because the optimum levels of inventories are related to the square root of the demand. Thus, if demand rises 20%, then the level of inventory will not increase 20%. This is in line with classical inventory theory and is evidence of the mathematical validity of the proposed fuzzy framework Harris (1913); Taft (1918).

The fuzzy inventory model also offers extra flexibility in decision making as it generates lower, modal and upper inventory quantities. The first, second, and third numbers are the pessimist, most likely, and optimist values, respectively. This can be useful for inventory managers to have because they can use it to understand how risky it is and make inventory decisions according to their goals and risk tolerances. For instance, companies dealing with more volatile markets will prefer to hold an inventory level closer to the max, whereas companies with more consistent demand patterns will prefer to hold an inventory level closer to the modal value.

Theoretically, the study has added the EOQ and EPQ concepts in a single fuzzy framework, and it has enriched the literature of inventory management. The earlier works have been mainly dedicated to either fuzzy EOQ models or fuzzy EPQ models separately (Gen, Tsujimura and Zheng, 1997; Kumar, 2023; Maharani, 2025). In the present research, both approaches are combined and it is shown that fuzzy arithmetic and defuzzification techniques can be applied in a consistent manner to both ordering and production inventory systems. The integration gives a complete methodology which can be used for different industrial applications.

The results are also in line with the latest advances in fuzzy inventory studies. According to Hemalatha and Annadurai (2023), fuzzy inventory models are more suitable than deterministic approaches because they are able to capture the uncertainty more effectively. Similarly, Rahaman, Mondal and Maiti (2022) revealed that fuzzy production inventory system is more effective in uncertain environment. Based on the findings of the present study, these conclusions can be drawn that fuzzy EOQ and EPQ models are more realistic inventory policies and still simple to compute.

On the managerial side, there are several benefits of the proposed model. First, it helps managers to make inventory decisions when they require an accurate estimate of the parameters when they are not possible. Second, it minimizes the likelihood of a shortage of stock due to unexpected fluctuations in requests for goods and services. Thirdly, it gives a structure to analyse inventory policies for various uncertainty scenarios. Further, the use of triangular fuzzy numbers makes the system easy to implement because just the lower, most likely and upper values of the fuzzy numbers are needed from the decision makers.

Although these benefits are noted, there are some drawbacks that should be recognized. The model assumes that there are no shortages and no lead time. Moreover, just triangular fuzzy numbers and the centroid defuzzification method are taken into account. In practice, the inventory systems can have more complexities including deterioration, multiple products, quantity discounts, inflation and stochastic lead times. Hence, the proposed model needs to be considered as an initial model that can be expanded to include more complex inventory characteristics.

Conclusion

In this study, an integrated fuzzy inventory model, combining the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models, is developed to tackle with the uncertainty in inventory management. In the traditional inventory models, the demand, ordering costs, holding costs and production rates are assumed to be known. But, in a real business where parameters related to inventory fluctuate and are not precise, such assumptions are unrealistic. In order to overcome these drawbacks, the fuzzy set theory was introduced in the modeling process of inventory and the uncertain parameters were represented by triangular fuzzy numbers.

This study started with a brief analysis of the theoretical background of the classical EOQ and EPQ models, and then discussed the function of fuzzy set theory in inventory optimization. Later, mathematical models for fuzzy EOQ and fuzzy EPQ models were constructed using fuzzy arithmetic and centroid based defuzzification method. This proposed framework will allow for representation of uncertain inventory parameters as ranges instead of a fixed value, which will more realistically capture inventory decision-making situations.

A numerical example was studied to show the applicability of the proposed methodology. The results indicated that the fuzzy inventory models yielded a bit more order and production quantities than their deterministic counterparts. The result indicates that the fuzzy models offer more cautious policies in inventory management, therefore, it can be useful to minimize the probability of stock-outs and operational problems due to uncertainty. In addition, the sensitivity analysis showed that the model developed is consistent with the inventory theory as optimal inventory levels were increasing with the rising demand level, but maintaining the same nonlinear relation.

The study helps in the inventory management literature by combining the EOQ and EPQ theories in the same framework with fuzzy. In contrast to most previous research which considers either ordering systems or production systems, the proposed model is a comprehensive one, which is able to consider both inventory systems simultaneously. The applicability and the simplicity of computation are also guaranteed by the use of triangular fuzzy numbers and centroid defuzzification.

In the managerial point of view, the developed fuzzy inventory model presents a valuable decision support tool for the organisations in uncertain business environments. The model can be used by managers to analyze inventory policies for various levels of uncertainty and build stronger inventory policies. The capacity to model uncertainty in demand and cost parameters may help to improve the accuracy of inventory planning and improve overall operations.

However, there are some limitations of this study. The model assumes that the inventory system is a single product, lead time is negligible and there are no shortage situations. The proposed framework can be further extended by considering the deterioration, shortages, quantity discounts, inflation effects, multi-items inventory system and using more sophisticated fuzzy numbers (trapezoidal or intuitionistic fuzzy numbers) in future research. In addition, hybrid systems incorporating fuzzy theory with machine learning and optimization methods for enhanced inventory decision-making in complex supply chain settings can also be investigated.

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