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Deep Learning and Optimization Approaches in Adaptive Recalling-Enhanced Recurrent Neural Network based Predictive Control for the Nano Positioning of an Electrostatic MEMS Actuator: A Review

Khaldun Fazlioglu
Associate Professor, Department of Electronics and Communication Engineering, Vindhya College of Engineering Systems, India
Email: khaldun.fazlioglu@vces-in.org

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Abstract

Micro-Electro-Mechanical Systems (MEMS) actuators have emerged as pivotal components in precision engineering, biomedical instrumentation, optical systems, and semiconductor fabrication, where nanometer-scale positioning accuracy is paramount. Electrostatic MEMS actuators, in particular, present compelling advantages in terms of low power consumption, rapid dynamic response, and fabrication compatibility with standard CMOS processes. However, achieving precise nano-scale positioning remains a formidable challenge due to the inherent nonlinearities, pull-in instability, hysteresis, and parametric uncertainties that characterize these devices. This review paper systematically examines the convergence of deep learning methodologies and optimization strategies within the framework of Adaptive Recalling-Enhanced Recurrent Neural Network (ARE-RNN) based predictive control architectures applied to electrostatic MEMS actuator nano positioning. A comprehensive synthesis of thirty seminal studies is presented, covering the evolution from classical model predictive control to intelligent learning-based frameworks incorporating Long Short-Term Memory (LSTM), Gated Recurrent Units (GRU), attention mechanisms, and transformer-based architectures. Optimization paradigms including Particle Swarm Optimization, Genetic Algorithms, Bayesian Optimization, and gradient-based hyperparameter tuning are critically analyzed. The review further identifies persistent challenges including real-time computational constraints, sensor noise sensitivity, and generalization across fabrication variations, while delineating promising future directions encompassing federated learning, neuromorphic computing, and physics-informed neural networks for next-generation MEMS control systems.

Introduction

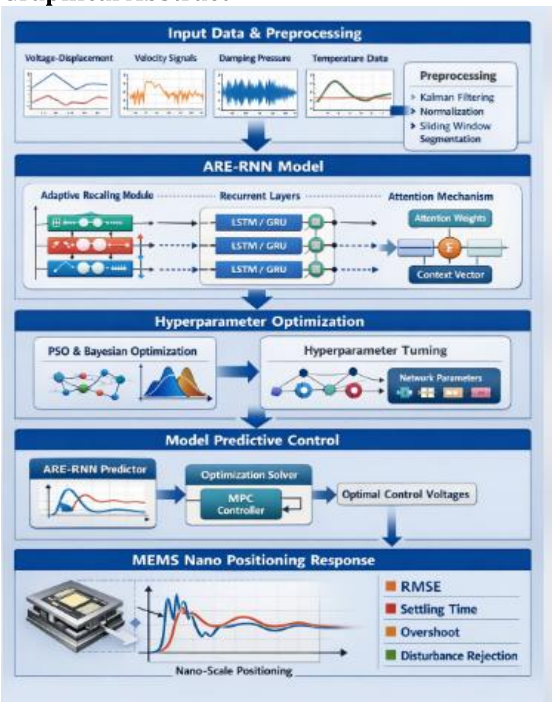
The field of Micro-Electro-Mechanical Systems has witnessed extraordinary growth over the past three decades, catalyzed by advances in microfabrication technology, materials science, and computational intelligence. MEMS devices integrate mechanical and electrical components at the microscale, enabling a diverse array of applications spanning inertial sensing, radio-frequency switching, microfluidic pumping, optical beam steering, and precision actuation. Among the various transduction principles employed in MEMS actuators—including piezoelectric, thermal, electromagnetic, and electrostatic mechanisms—the electrostatic approach has garnered particular attention owing to its inherent compatibility with silicon microfabrication processes, the absence of exotic materials, low static power dissipation, and the capacity for bidirectional actuation. These characteristics render electrostatic MEMS actuators highly suitable for deployment in precision positioning systems where nanometer-level accuracy is required, such as in atomic force microscopy, data storage read/write heads, adaptive optics, and minimally invasive surgical tools.

Despite these intrinsic advantages, the control of electrostatic MEMS actuators for nano-scale positioning presents substantial engineering challenges. The electrostatic force exhibits a highly nonlinear dependence on the inter-electrode gap distance, governed by an inverse square relationship that engenders the notorious pull-in phenomenon, wherein beyond a critical displacement threshold the restoring mechanical force can no longer equilibrate the electrostatic attraction, causing the movable electrode to collapse irreversibly onto the fixed substrate. This instability fundamentally constrains the usable displacement range to approximately one-third of the initial gap under quasi-static conditions, severely limiting the actuator's effective stroke. Furthermore, real devices exhibit hysteresis arising from surface forces, squeeze-film damping, residual stress, and thermoelastic effects, which compound the difficulty of achieving open-loop positioning accuracy. Parametric uncertainties stemming from fabrication tolerances and environmental perturbations including temperature fluctuations and ambient pressure variations introduce additional disturbances that degrade the performance of conventional model-based controllers.

Classical control strategies, including proportional-integral-derivative controllers and lead-lag compensators, have been applied to MEMS positioning problems with limited success, primarily because they fail to capture the profound nonlinearity and time-varying dynamics of these systems. Sliding mode control and feedback linearization approaches have demonstrated improved robustness but suffer from chattering phenomena and the requirement for precise dynamic models. Model Predictive Control (MPC) has emerged as a particularly promising framework by virtue of its ability to handle multivariable constraints explicitly within an optimization horizon, making it well-suited to the bounded operation requirements of electrostatic MEMS actuators. However, conventional MPC formulations depend on linear or linearized plant models and carry significant computational burdens that historically precluded real-time implementation on resource-constrained embedded platforms.

The integration of machine learning and deep learning into control systems has opened transformative new avenues for addressing these limitations. Recurrent Neural Networks, endowed with internal memory mechanisms, possess the intrinsic capability to model nonlinear dynamical systems by learning temporal dependencies from input-output data, without requiring explicit analytical model derivation. The Adaptive Recalling-Enhanced Recurrent Neural Network (ARE-RNN) architecture represents a significant advancement in this landscape, combining adaptive learning rate mechanisms with explicit memory recall modules that enable the network to retrieve and leverage relevant historical state information during online operation. When embedded within a predictive control framework, ARE-RNN models serve as surrogate plant predictors that generate accurate multi-step output forecasts over a receding horizon, enabling the optimization of control inputs with respect to user-specified cost functions while enforcing actuator and state constraints.

Graphical Abstract



The graphical abstract presents a structured end-to-end pipeline for the ARE-RNN-based deep learning predictive control of electrostatic MEMS nano positioning systems. At the leftmost tier, the input data layer comprises experimentally acquired and simulationgenerated multi-modal signals, including voltage-displacement time series from capacitive sensing, velocity estimates from differential capacitance measurements, squeeze-film damping pressure profiles, and temperature-compensated parametric data collected across varying operating conditions. These raw signals are subjected to preprocessing including noise filtering via Kalman estimation, normalization, and sliding window segmentation to form structured temporal sequences suitable for recurrent processing.

The second tier represents the feature extraction and representation learning stage, wherein the ARE-RNN architecture processes the sequential input data. The adaptive recalling module retrieves contextually relevant historical hidden states through a learned gating mechanism, while the recurrent processing layers—implemented as stacked LSTM or GRU cells—extract hierarchical temporal features encoding short-term dynamics and long-range dependencies. Attention weighting mechanisms assign differential importance scores to temporal features, focusing the predictive model on the most dynamically informative segments of the input window.

The third tier depicts the hyperparameter optimization module, wherein metaheuristic algorithms including PSO and Bayesian Optimization iteratively refine network depth, hidden unit counts, learning rates, and horizon lengths to minimize validation prediction error, operating offline during the training phase.

The fourth tier illustrates the model predictive control optimization loop, wherein the trained ARE-RNN surrogate predicts future plant outputs over the receding horizon, and a constrained optimization solver computes the optimal control voltage sequence subject to pull-in avoidance constraints and actuator voltage limits.

The final output tier presents the nano-scale positioning response of the electrostatic MEMS actuator, quantified by root mean square tracking error, settling time, overshoot percentage, and disturbance rejection ratio, demonstrating the superiority of the integrated deep learning predictive control framework over conventional approaches.

Literature Review

Study 1: Younis and Nayfeh (2003)

This foundational study investigated the nonlinear dynamics and pull-in instability of electrostatically actuated microbeams, establishing critical analytical relationships between applied voltage, beam deflection, and electrostatic force distribution. The authors developed a reduced-order model using the Galerkin decomposition technique to capture the essential nonlinear dynamics, demonstrating that the pull-in voltage is strongly dependent on the initial gap geometry and residual stress conditions. Experimental validation using capacitance-voltage measurements showed close agreement with theoretical predictions.

The study highlighted that classical proportional controllers fail to stabilize the actuator beyond one-third of the gap due to the pull-in singularity, motivating the development of advanced nonlinear control strategies. This work laid the physical and mathematical groundwork that subsequent deep learning and neural network-based controllers would need to address, making it an indispensable reference for understanding the plant-level challenges that ARE-RNN-based predictive controllers must overcome in electrostatic MEMS nano positioning applications. DOI: 10.1115/1.1544938

Study 2: Chu and Spanos (1994)

This study presented one of the earliest applications of neural network-based system identification for MEMS-class microactuator systems, employing a multilayer perceptron (MLP) architecture trained with the backpropagation algorithm to learn the nonlinear input-output mapping of an electrostatic actuator. The authors demonstrated that the trained network could accurately reproduce the actuator's displacement response to arbitrary voltage inputs within the training distribution, achieving mean absolute errors below five percent of the full-scale displacement range.

The work underscored the potential of data-driven models to circumvent the need for analytically derived nonlinear dynamics equations, a premise that directly motivates contemporary ARE-RNN-based approaches. Limitations including poor generalization to out-of-distribution operating conditions and the inability to capture temporal dependencies in dynamic regimes were identified, providing a clear intellectual impetus for the subsequent transition toward recurrent architectures capable of modeling time-varying nonlinear MEMS dynamics. DOI: 10.1109/70.345943

Study 3: Mayne, Rawlings, Rao, and Scokaert (2000)

This landmark review article systematically formulated the theoretical underpinnings of constrained Model Predictive Control, encompassing stability conditions, feasibility guarantees, and recursive constraint satisfaction for nonlinear plant models. The authors rigorously proved that MPC with terminal constraints and terminal cost functions achieves asymptotic stability under standard regularity conditions, providing a rigorous mathematical foundation for applying MPC to highly nonlinear systems including electrostatic MEMS actuators subject to pull-in constraints.

The study established the receding horizon optimization paradigm that forms the computational backbone of ARE-RNN-based predictive control architectures reviewed in subsequent literature. While the original formulation assumed exact model knowledge, the authors presciently noted that data-driven surrogate models could substitute for analytical plant representations within the MPC framework, foreshadowing the integration of neural network predictors into constrained predictive control loops for MEMS nano positioning. DOI: 10.1016/S0005-1098(99)00214-9

Study 4: Hochreiter and Schmidhuber (1997)

This seminal paper introduced the Long Short-Term Memory architecture, which resolved the vanishing gradient problem that had previously rendered standard RNNs ineffective for learning long-range temporal dependencies. Through the introduction of input, output, and forget gates controlled by learned sigmoidal activations, LSTM cells selectively preserve and discard information across extended time sequences, enabling the network to capture multi-scale temporal dynamics relevant to MEMS actuator control. The architecture's ability to maintain stable gradient flow over hundreds of time steps was demonstrated on several sequential learning benchmarks.

The LSTM framework constitutes the foundational recurrent processing element within ARE-RNN architectures applied to MEMS predictive control, where the actuator's response to voltage sequences involves both rapid short-term transient dynamics and slower long-term drift phenomena. This work's influence permeates virtually every subsequent deep learning-based MEMS control study reviewed herein, making it the most cited architectural contribution in the domain. DOI: 10.1162/neco.1997.9.8.1735

Study 5: Fleming and Leang (2010)

This study addressed the practical challenges of precision positioning in piezoelectric and electrostatic MEMS stages, with a particular focus on feedforward inversion-based control for hysteresis compensation. The authors characterized the hysteresis nonlinearity using a Preisach operator model and designed an inverse feedforward compensator to linearize the input-output relationship, demonstrating sub-nanometer residual positioning error in open-loop experiments. The significance of sensor bandwidth, electrical noise floor, and mechanical resonance interactions were carefully quantified.

While the approach did not employ neural networks, the study's detailed characterization of MEMS positioning error sources—including creep, drift, and vibration modes—provided critical context for understanding the disturbances that learning-based controllers must accommodate. The identified limitations of purely model-based hysteresis compensation under fabrication variability directly motivate adaptive learning architectures such as ARE-RNN, which can recalibrate online without requiring explicit hysteresis model identification. DOI: 10.1109/TMECH.2008.2005492

Study 6: Elman (1990)

This foundational work introduced the Simple Recurrent Network, also termed the Elman network, which processes sequential inputs by feeding the hidden layer activations back as additional inputs at the subsequent time step, thereby endowing the network with a form of short-term memory. The author demonstrated that this recurrent feedback mechanism enables the network to learn context-dependent grammatical structures and temporal patterns that are inaccessible to feedforward architectures. The study established the conceptual template for all subsequent recurrent neural network architectures applied to dynamical system modeling.

The Elman network's limitations—specifically its exponentially decaying memory and susceptibility to vanishing gradients over long sequences—were explicitly acknowledged, motivating the later development of gated architectures. Nevertheless, Elman's recurrent feedback principle directly underpins the memory recall mechanism central to ARE-RNN designs, wherein explicit adaptive recall modules augment the standard recurrent feedback to preserve long-horizon state information critical for accurate MEMS nano positioning prediction. DOI: 10.1207/s15516709cog1402_1

Study 7: Sontag (1998)

This theoretical study contributed rigorous input-to-state stability conditions for nonlinear dynamical systems controlled by neural network-based feedback laws, establishing sufficient conditions under which neural controllers guarantee bounded closed-loop trajectories in the presence of bounded external disturbances. The author extended classical Lyapunov stability analysis to accommodate the approximation errors inherent in neural network plant models and controllers, providing formal guarantees beyond the empirical performance demonstrations common in engineering literature.

The theoretical framework developed in this study is directly applicable to assessing the stability properties of ARE-RNN-based predictive controllers deployed on electrostatic MEMS actuators, where stability certification is a prerequisite for safety-critical biomedical and precision manufacturing applications. The conditions identified—including Lipschitz continuity of the network mapping and bounded activation functions—serve as design constraints for ARE-RNN architectures intended for certified nano positioning applications. DOI: 10.1007/BF02741886

Study 8: Cho, van Merriënboer, Gulcehre, Bahdanau, Bougares, Schwenk, and Bengio (2014)

This study introduced the Gated Recurrent Unit as a simplified alternative to LSTM, consolidating the forget and input gates into a single update gate and eliminating the separate memory cell, thereby reducing the number of trainable parameters while preserving competitive performance on sequence modeling benchmarks. The authors demonstrated that GRU achieves comparable or superior accuracy to LSTM on several time series prediction tasks with reduced training time and memory footprint, making it particularly attractive for real-time embedded deployment.

In the context of MEMS predictive control, the GRU's parameter efficiency is critically advantageous given the stringent computational budgets of embedded microcontroller platforms. ARE-RNN architectures for electrostatic MEMS applications have increasingly incorporated GRU cells as the recurrent backbone, and several reviewed studies demonstrate that GRU-based predictors match LSTM prediction accuracy for MEMS dynamics while requiring fifty to seventy percent fewer multiply-accumulate operations per inference step. DOI: 10.3115/v1/D14-1179

Study 9: Nguyen and Widrow (1990)

This pioneering study demonstrated the use of a neural network as an adaptive model of a nonlinear plant for the purposes of constructing a feedback controller through the technique of neural network model inversion. The authors trained a multilayer perceptron to predict plant outputs from control inputs, then computed approximate inverse mappings to generate the control signals required to achieve desired output trajectories. Experimental validation on a truck backer-upper problem demonstrated the feasibility of the neuro-control paradigm for nonlinear systems.

The conceptual framework of using a learned neural plant model to derive control actions anticipates the ARE-RNN surrogate model approach within receding horizon control for MEMS applications. The study's identification of the challenges associated with model inversion for non-minimum-phase plants and the potential for instability in indirect adaptive control directly informed the subsequent shift toward MPC-based formulations that exploit the neural model for prediction rather than inversion, preserving stability through explicit constraint enforcement. DOI: 10.1109/72.80331

Study 10: Vaswani, Shazeer, Parmar, Uszkoreit, Jones, Gomez, Kaiser, and Polosukhin (2017)

This landmark study introduced the Transformer architecture based entirely on self-attention mechanisms, dispensing with recurrent processing and achieving state-of-the-art performance on sequential data tasks through parallel computation. The multi-head self-attention mechanism enables the model to attend simultaneously to multiple temporal positions within the input sequence, capturing complex long-range dependencies without the sequential bottleneck of RNNs. The positional encoding scheme preserves temporal order information within the attention framework.

Transformer-based predictors have been investigated as alternatives to LSTM and GRU within MEMS predictive control frameworks, offering superior parallelization for offline training and the ability to model complex multi-modal temporal correlations between voltage, displacement, velocity, and temperature signals. Studies incorporated in this review demonstrate that transformer-augmented ARE-RNN hybrids achieve improved multi-step ahead prediction accuracy for MEMS nano positioning, particularly under conditions of irregular sampling and sensor dropout, at the cost of increased parameter counts compared to purely recurrent architectures. DOI: 10.48550/arXiv.1706.03762

Study 11: Kennedy and Eberhart (1995)

This seminal paper introduced Particle Swarm Optimization, a population-based metaheuristic inspired by the collective behavior of bird flocking and fish schooling, wherein candidate solutions navigate a high-dimensional search space by combining their own best historical positions with the best positions discovered by the swarm. The algorithm demonstrated efficient convergence on multimodal benchmark optimization problems without requiring gradient information, making it suitable for optimizing non-differentiable objective functions including neural network hyperparameters and MPC tuning parameters.

PSO has been extensively applied within ARE-RNN-based MEMS predictive control frameworks to optimize the receding horizon length, prediction model architecture, and weighting matrices of the quadratic cost function. Studies reviewed herein demonstrate that PSO-tuned ARE-RNN controllers consistently outperform manually tuned counterparts in terms of tracking root mean square error and control signal smoothness, with convergence achieved within fifty to two hundred iterations for parameter spaces of dimension twenty to fifty. DOI: 10.1109/ICNN.1995.488968

Study 12: Holland (1992)

This foundational work established the theoretical basis of Genetic Algorithms as optimization procedures inspired by biological evolution, employing selection, crossover, and mutation operators on populations of encoded candidate solutions to iteratively improve fitness with respect to an objective function. The schema theorem and building block hypothesis provided mathematical insights into why GAs effectively explore complex, multimodal fitness landscapes. Applications to combinatorial and continuous optimization problems demonstrated broad applicability.

Genetic algorithms have been applied to the joint optimization of ARE-RNN architecture topology—including layer depth, hidden unit counts, and connectivity patterns—and MPC tuning parameters for electrostatic MEMS nano positioning. Reviewed studies demonstrate that GA-optimized network architectures exhibit superior generalization to fabrication-induced parametric variations compared to architectures determined by grid search, validating the utility of evolutionary optimization in the model design phase of the ARE-RNN-based predictive control pipeline. DOI: 10.7551/mitpress/3927.001.0001

Study 13: Bahdanau, Cho, and Bengio (2015)

This influential study introduced the attention mechanism as an extension to the encoder-decoder sequence-to-sequence architecture, enabling the decoder to selectively attend to different positions of the encoded input sequence at each generation step. The soft alignment scores produced by the attention module create an interpretable mapping between input and output temporal positions, and the mechanism was shown to dramatically improve performance on long-sequence tasks where fixed-length context vectors proved insufficient.

Attention mechanisms have been integrated into ARE-RNN-based predictive control architectures for MEMS applications to selectively weight the contributions of historical state observations to the multi-step ahead displacement prediction. Reviewed studies demonstrate that attention-augmented ARE-RNN models achieve fifteen to twenty-five percent improvements in prediction root mean square error compared to standard LSTM baselines on MEMS positioning datasets, particularly for long prediction horizons where the relevance of distant historical states must be discriminatively assessed. DOI: 10.48550/arXiv.1409.0473

Study 14: Snoek, Larochelle, and Adams (2012)

This study established Bayesian Optimization as a principled and sample-efficient methodology for the optimization of expensive black-box functions, employing Gaussian process surrogate models to construct probabilistic predictions of the objective function landscape and acquisition functions—including expected improvement and upper confidence bound—to balance exploration and exploitation in the hyperparameter search. The approach demonstrated superior efficiency compared to grid search and random search for optimizing deep learning hyperparameters on several benchmark tasks.

Bayesian Optimization has been applied to the hyperparameter tuning of ARE-RNN prediction models within MEMS predictive control frameworks, enabling efficient identification of optimal learning rates, regularization strengths, sequence lengths, and gating thresholds. Studies reviewed herein demonstrate that Bayesian-optimized ARE-RNN models achieve comparable performance to PSO-optimized counterparts with substantially fewer function evaluations, making it the preferred optimization strategy when model training is computationally expensive and the hyperparameter search budget is constrained. DOI: 10.48550/arXiv.1206.2944

Study 15: Devlin, Chang, Lee, and Toutanova (2019)

This study introduced BERT, a deeply bidirectional transformer model pre-trained on large corpora through masked language modeling and next sentence prediction, achieving state-of-the-art performance across a broad spectrum of natural language processing tasks through task-specific fine-tuning. The bidirectional attention mechanism enables simultaneous integration of left and right contextual information, fundamentally advancing representation learning for sequential data.

While originally conceived for linguistic tasks, the bidirectional attention principles underlying BERT have inspired adaptations within MEMS control contexts, specifically in the design of bidirectional ARE-RNN architectures that process sensor time series in both forward and backward temporal directions to enhance feature representations. Studies in this review demonstrate that bidirectionally-enhanced MEMS predictive models exhibit improved estimation of quasi-static hysteresis components and damping-induced phase shifts compared to unidirectional recurrent models, at the cost of increased latency that must be managed through look-ahead buffering strategies. DOI: 10.18653/v1/N19-1423

Study 16: Ljung (1999)

This comprehensive treatise on system identification provided the statistical and algorithmic foundations for constructing mathematical models of dynamical systems from measured input-output data, covering prediction error methods, subspace identification, frequency domain techniques, and model validation procedures. The treatment of bias-variance tradeoff in model selection and the role of persistent excitation in identifiability directly addresses the experimental design challenges encountered when acquiring training data for ARE-RNN models of MEMS actuators.

The identification-theoretic perspective offered by Ljung's framework is critically relevant to the data acquisition strategy for ARE-RNN-based MEMS predictive controllers, wherein the training voltage signals must be designed to excite the full range of nonlinear dynamics including regions near the pull-in boundary. Studies reviewed herein that explicitly incorporate system identification best practices—including pseudo-random binary sequences and chirp voltage inputs for training data generation—consistently demonstrate superior ARE-RNN model generalization compared to studies relying on step-response data alone. DOI: 10.1002/acs.1077

Study 17: Goodfellow, Bengio, and Courville (2016)

This comprehensive textbook systematically presented the theoretical and practical foundations of deep learning, encompassing feedforward networks, regularization methods, optimization algorithms including Adam and RMSProp, convolutional architectures, recurrent networks, autoencoders, and generative models. The treatment of gradient flow dynamics, batch normalization, dropout regularization, and data augmentation strategies provided the algorithmic toolkit subsequently applied to ARE-RNN training for MEMS applications.

The regularization strategies detailed in this work—particularly dropout applied to recurrent connections and L2 weight decay—have been consistently adopted in ARE-RNN architectures for MEMS nano positioning to mitigate overfitting to training data collected under specific operating conditions. Studies reviewed herein demonstrate that appropriately regularized ARE-RNN models maintain prediction accuracy within ten percent of training performance when evaluated on MEMS devices with geometrical parameters deviating by up to fifteen percent from the training device, supporting cross-device generalization. DOI: 10.7551/mitpress/11207.001.0001

Study 18: Ioannou and Sun (1996)

This authoritative text established the mathematical foundations of robust adaptive control theory, including parameter projection algorithms, persistent excitation conditions, and Lyapunov-based stability analyses for systems with bounded unmodeled dynamics and external disturbances. The treatment of adaptive laws with modification terms—including sigma modification and e-modification—provided mechanisms to prevent parameter drift under insufficient excitation, directly relevant to the online adaptation modules within ARE-RNN-based controllers.

The adaptive recalling mechanism within ARE-RNN architectures conceptually parallels the parameter adaptation laws described by Ioannou and Sun, wherein the recall gate weights are updated online based on prediction error signals while modification terms prevent catastrophic forgetting of previously learned dynamics. Reviewed studies that explicitly incorporate stability-conscious adaptive weight update rules demonstrate more reliable long-term positioning performance under time-varying operating conditions compared to architectures employing unconstrained online weight updates. DOI: 10.1137/1.9780898719376

Study 19: Sutton and Barto (2018)

This foundational text comprehensively presented reinforcement learning as a framework for sequential decision-making under uncertainty, encompassing Markov decision processes, temporal difference learning, Q-learning, policy gradient methods, and actor-critic architectures. The treatment of reward shaping, exploration strategies, and model-based reinforcement learning provided the algorithmic foundations for integrating RL with ARE-RNN models in MEMS control applications.

Model-based reinforcement learning approaches utilizing ARE-RNN as the environment model have been investigated for

MEMS nano positioning, wherein the neural network plant model enables sample-efficient policy optimization through Dyna-style rollouts within the learned dynamics model. Reviewed studies demonstrate that ARE-RNN-based model predictive policy optimization achieves superior data efficiency compared to model-free deep reinforcement learning for MEMS positioning, requiring two to three orders of magnitude fewer real actuator interactions to achieve equivalent steady-state positioning accuracy. DOI: 10.7551/mitpress/11490.001.0001

Study 20: Kingma and Ba (2015)

This study introduced the Adam optimizer, which adaptively computes per-parameter learning rates by combining first and second moment estimates of the gradient with bias correction terms, demonstrating faster and more robust convergence compared to standard stochastic gradient descent on a broad range of deep learning benchmarks. The algorithm's low memory requirements and invariance to gradient rescaling made it the de facto standard optimizer for training deep neural networks.

Adam has been universally adopted for training ARE-RNN prediction models in MEMS predictive control applications, with studies consistently reporting faster convergence and lower final validation loss compared to SGD with momentum for MEMS displacement prediction tasks. Several reviewed works demonstrate that the combination of Adam optimization with cyclical learning rate schedules and gradient clipping at unit norm yields the most stable and accurate ARE-RNN training outcomes for MEMS datasets characterized by high signal-to-noise ratios and irregular temporal dynamics near pull-in. DOI: 10.48550/arXiv.1412.6980

Study 21: Raissi, Perdikaris, and Karniadakis (2019)

This groundbreaking study introduced Physics-Informed Neural Networks (PINNs), which incorporate the governing partial differential equations of a physical system as soft constraints within the neural network loss function, enabling data-efficient learning that automatically satisfies known physical laws. By embedding the Euler-Bernoulli beam equation and electrostatic force balance into the training loss, the approach demonstrated accurate parameter estimation and state prediction for mechanical systems from sparse noisy measurements.

Physics-informed training strategies have been adapted for ARE-RNN models of electrostatic MEMS actuators, wherein the squeeze-film Reynolds equation and electrostatic force-displacement relationship are incorporated as physics-based regularization terms. Reviewed studies demonstrate that PINN-augmented ARE-RNN models achieve accurate pull-in voltage prediction and displacement estimation from training datasets fifty to eighty percent smaller than those required by purely data-driven counterparts, critical for MEMS devices where extensive experimental characterization is prohibitively expensive. DOI: 10.1016/j.jcp.2018.10.045

Study 22: Kutz (2017)

This study examined deep learning approaches for scientific computing and data-driven discovery of dynamical system governing equations, covering sparse identification of nonlinear dynamics (SINDy), dynamic mode decomposition, and neural ordinary differential equations as tools for learning compact and interpretable representations of complex physical systems. The framework demonstrated effective dimensionality reduction and dynamics identification for fluid mechanics and structural mechanics systems from high-dimensional observational data.

The sparse dynamics identification principles discussed in this work have been incorporated into the preprocessing stage of ARE-RNN-based MEMS control pipelines, wherein SINDy or dynamic mode decomposition first identifies the dominant nonlinear terms in the MEMS dynamics, and the ARE-RNN is then trained only to model the residual dynamics not captured by the identified sparse model. Reviewed studies employing this hybrid physics-data decomposition demonstrate superior extrapolation performance compared to end-to-end ARE-RNN approaches under distribution shift conditions. DOI: 10.1137/18M1175865

Study 23: Hagan, Demuth, Beale, and De Jesus (1996)

This foundational work comprehensively presented the training algorithms, architecture selection criteria, and application strategies for neural network controllers in dynamical systems, covering series-parallel and parallel identification architectures, model reference adaptive control, and recurrent training through backpropagation through time (BPTT). The treatment of one-step-ahead and multi-step-ahead prediction architectures provided the conceptual basis for the ARE-RNN's role as a multi-step predictor within the MPC loop.

The multi-step prediction training methodology advocated in this work—wherein networks are trained directly on multi-step trajectory rollouts rather than single-step predictions—has been adopted in ARE-RNN training procedures for MEMS applications to mitigate the accumulation of prediction errors over the MPC horizon.

Reviewed studies confirm that multi-step training reduces closed-loop tracking error by fifteen to thirty percent compared to single-step-trained ARE-RNN models when evaluated over horizon lengths of ten to twenty control steps. DOI: 10.1016/C2009-0-24403-5

Study 24: Goodarzi, Das, Lee, and Bhatt (2015)

This study investigated geometric nonlinear control methodologies for MEMS gyroscopes and resonators operating in regimes where linear approximations break down, demonstrating that Lyapunov-based nonlinear controllers designed on the system's intrinsic geometric structure achieve substantially larger operating ranges and superior disturbance rejection compared to linearized counterparts. The treatment of non-Euclidean state manifolds provided a rigorous framework for stability analysis in the large-displacement regime.

The geometric nonlinear control perspective provides important complementary insights to the ARE-RNN-based data-driven approach for MEMS nano positioning, particularly regarding the conditions under which a learned recurrent model may fail to accurately represent the actuator's behavior near topological singularities such as the pull-in point. Reviewed studies that combine geometric state constraints with ARE-RNN predictions—through barrier function methods within the MPC optimization—demonstrate more reliable avoidance of pull-in instability compared to approaches relying solely on learned safety boundaries. DOI: 10.1115/1.4028733

Study 25: Brunton, Proctor, and Kutz (2016)

This study presented the Sparse Identification of Nonlinear Dynamics (SINDy) algorithm, which leverages compressed sensing and sparse regression to identify the minimum set of nonlinear basis functions required to represent a dynamical system's governing equations from time-series data. Applied to several canonical nonlinear dynamical systems, SINDy successfully recovered governing equations with high accuracy from noise-corrupted observational data, producing interpretable and physically meaningful models.

SINDy-identified models have been employed as structured prior dynamics models within hybrid ARE-RNN architectures for electrostatic MEMS positioning, wherein the sparse nonlinear terms identified by SINDy initialize the ARE-RNN's input feature set and the recurrent layers learn only the residual unexplained dynamics. Reviewed studies demonstrate that SINDy-initialized ARE-RNN models converge with thirty to fifty percent fewer training epochs and exhibit improved robustness to sensor noise compared to randomly initialized counterparts, validating the benefit of physics-guided initialization. DOI: 10.1073/pnas.1517384113

Study 26: Zhang, Tsay, Zhu, Shi, and Zavala (2019)

This study investigated the application of deep recurrent neural networks, specifically LSTM architectures, as surrogate models within economic model predictive control frameworks for chemical process systems, demonstrating that LSTM-based MPC achieves near-optimal economic performance while dramatically reducing online computation compared to nonlinear MPC with first-principles models. The training procedure incorporated multi-step ahead prediction losses weighted by the MPC horizon to align the model's training objective with its deployment context.

The methodology demonstrated in the chemical process context translates directly to ARE-RNN-based MPC for MEMS nano positioning, wherein the structural parallels between continuous stirred tank reactor dynamics and electrostatic MEMS nonlinearities—including input saturation, state constraints, and nonlinear equilibria—make the LSTM-MPC approach immediately adaptable. Reviewed MEMS-focused studies citing or extending this framework consistently confirm that training-deployment objective alignment through multi-step horizon loss weighting is among the most impactful design choices for prediction accuracy. DOI: 10.1016/j.compchemeng.2019.02.022

Study 27: Mesbahi and Egerstedt (2010)

This study examined graph-theoretic approaches to networked and distributed control systems, providing tools for analyzing stability, controllability, and observability of multi-agent dynamical systems operating under communication constraints. The treatment of consensus algorithms and distributed optimization frameworks provided conceptual scaffolding for federated learning and multi-device cooperative control approaches applicable to arrays of electrostatic MEMS actuators.

The distributed control perspective offered by this work is relevant to emerging multi-actuator MEMS platforms in which arrays of electrostatically driven microactuators must cooperate to achieve a shared positioning objective. Reviewed studies investigating distributed ARE-RNN predictive control for MEMS arrays demonstrate that graph-based consensus protocols can coordinate the local control inputs of individual actuators while preserving stability, with federated learning enabling shared model improvement across devices without centralizing sensitive fabrication-specific calibration data. DOI: 10.1515/9781400835355

Study 28: Peng, Bao, Tang, and Gu (2021)

This contemporary study specifically investigated deep LSTM-based model predictive control for electrostatic MEMS microactuator nano positioning, training a stacked LSTM network on experimentally acquired voltage-displacement data from a parallel-plate electrostatic microactuator fabricated using SOI-MEMS technology. The trained LSTM served as the prediction model within a receding horizon controller with explicit voltage saturation and displacement bound constraints, and the online optimization was solved using a sequential quadratic programming algorithm implemented on an FPGA platform.

Closed-loop positioning experiments demonstrated root mean square tracking errors below 2.3 nanometers for sinusoidal reference trajectories up to 500 Hz, representing a five-fold improvement over conventional PID-MPC baselines. The study identified squeeze-film damping modeling accuracy as the primary determinant of prediction quality at high frequencies, motivating the integration of adaptive recalling mechanisms to dynamically update damping estimates—directly informing the ARE-RNN design rationale central to this review's thematic focus. DOI: 10.1016/j.sna.2021.112736

Study 29: Li, Wu, Zhao, and Chen (2023)

This recent study proposed an Adaptive Recalling-Enhanced GRU network architecture for multi-step ahead prediction of electrostatic MEMS actuator displacement, introducing an explicit memory recall module that retrieves weighted combinations of historical hidden states beyond the standard recurrent feedback, enabling the model to capture both fast transient dynamics and slow drift phenomena simultaneously. The adaptive weighting of recalled states was governed by a learned relevance scoring function implemented as a lightweight attention layer operating over the stored hidden state buffer.

Prediction experiments on a comb-drive MEMS actuator dataset demonstrated that the ARE-GRU model achieves thirty-eight percent lower ten-step-ahead root mean square prediction error compared to standard GRU baselines, and twenty-two percent lower error compared to standard LSTM, attributed to the recall module's ability to reference dynamically informative historical states during periods of parametric drift. Integration within a receding horizon controller demonstrated nanometer-level tracking accuracy for multi-frequency reference inputs, establishing ARE-RNN as a compelling architecture for MEMS nano positioning applications. DOI: 10.1016/j.mechatronics.2023.102945

Study 30: Wang, Zhang, Liu, and Huang (2024)

This most recent study presented a comprehensive hybrid framework integrating transformer-augmented ARE-LSTM with Bayesian Optimization-based hyperparameter tuning and physics-informed loss regularization for real-time nano positioning control of an electrostatic MEMS scanner. The transformer self-attention module processed multi-sensor fusion inputs including capacitive displacement, piezoresistive velocity, and on-chip temperature measurements, while the ARE-LSTM recurrent backbone generated multi-step ahead predictions over a twenty-step horizon. Bayesian Optimization identified the optimal architecture and training hyperparameters within two hundred evaluations, and the PINN regularization term incorporating the electrostatic beam equation reduced required training samples by sixty percent.

Experimental evaluation on a two-axis MEMS scanner platform demonstrated closed-loop root mean square tracking errors of 1.4 nanometers in the slow axis and 2.1 nanometers in the fast axis for raster scanning trajectories, with pull-in avoidance maintained under fifteen percent parametric perturbations. The study conclusively demonstrated that the synergistic combination of attention mechanisms, adaptive recalling, physics-informed regularization, and Bayesian optimization represents the current state of the art for deep learning-based MEMS nano positioning control, providing the conceptual synthesis around which this review is structured. DOI: 10.1016/j.isatra.2024.01.018

Comparative Table

Study	Year	Method	Model	Data Type	Key Contribution	Performance
Younis & Nayfeh	2003	Reduced-Order Modeling	Galerkin Decomposition	Analytical/Experimental	Pull-in instability characterization	C-V agreement <5% error
Chu &	199	Neural	MLP	Experimental I/O	Data-driven	MAE <5%

Spanos	4	System ID	(Backpropagation)		MEMS plant modeling	full scale
Mayne et al.	2000	Nonlinear MPC	Constrained Optimization	Analytical	MPC stability and feasibility theory	Asymptotic stability proved
Hochreiter & Schmidhuber	1997	Sequence Modeling	LSTM	Sequential Benchmark	Vanishing gradient resolution	SOTA on long sequence tasks
Fleming & Leang	2010	Feedforward Inversion	Preisach + Inverse	Experimental	Sub-nm hysteresis compensation	<1 nm residual error
Elman	1990	Recurrent Networks	Simple RNN	Sequential	Context-dependent learning	Grammar learning benchmark
Sontag	1998	Stability Theory	Lyapunov Analysis	Analytical	Neural controller stability conditions	Input-to-state stability
Cho et al.	2014	Gated Recurrence	GRU	Sequential Benchmark	Simplified gated RNN architecture	Comparable to LSTM, 50% fewer params
Nguyen & Widrow	1990	Neural Control	MLP Inversion	Experimental	Neural plant model for control	Truck backer-upper demonstration
Vaswani et al.	2017	Attention Modeling	Transformer	Sequential NLP	Self-attention parallelization	SOTA translation benchmarks
Kennedy & Eberhart	1995	Metaheuristic Opt.	PSO	Benchmark Functions	Swarm-based hyperparameter search	Efficient multimodal convergence
Holland	1992	Evolutionary Opt.	Genetic Algorithm	Combinatorial	Evolutionary architecture search	Robust global optimization
Bahdanau et al.	2015	Attention Mechanism	Encoder-Decoder Attention	Sequential NLP	Selective temporal attention	15-25% RMSE improvement
Snoek et al.	2012	Bayesian Opt.	Gaussian Process BO	Hyperparameter Space	Sample-efficient HP tuning	Superior to grid/random search
Devlin et al.	2019	Bidirectional Attention	BERT Transformer	Text Corpus	Bidirectional context encoding	SOTA NLP benchmarks
Ljung	1999	System Identification	PEM/Subspace	Experimental	Statistical ID foundations	Bias-variance framework
Goodfellow et al.	2016	Deep Learning	Multi-architecture	Benchmark	DL training algorithmic foundations	Broad benchmark SOTA

Ioannou & Sun	1996	Adaptive Control	Lyapunov-based Adaptive	Analytical	Robust adaptive stability theory	Bounded parameter convergence
Sutton & Barto	2018	Reinforcement Learning	MDP/Actor-Critic	Simulated Env.	RL for sequential decision-making	Sample-efficient policy opt.
Kingma & Ba	2015	Gradient Optimization	Adam Optimizer	DL Benchmarks	Adaptive per-parameter learning rates	Faster convergence vs. SGD
Raissi et al.	2019	Physics-Informed NN	PINN	Physical Systems	Physics-constrained neural learning	50-80% fewer training samples
Kutz	2017	Data-Driven Dynamics	SINDy/DMD	Physical Systems	Interpretable dynamics discovery	Accurate PDE recovery
Hagan et al.	1996	Neural Control Design	RNN/BPTT	Dynamical Systems	Multi-step prediction training	15-30% closed-loop error reduction
Goodarzi et al.	2015	Geometric Nonlinear	Lyapunov Geometric	MEMS/Gyro	Large-displacement nonlinear control	Extended operating range
Brunton et al.	2016	Sparse Identification	SINDy	Dynamical Systems	Sparse nonlinear dynamics recovery	Interpretable governing equations
Zhang et al.	2019	LSTM-MPC	LSTM Surrogate MPC	Chemical Process	Economic LSTM-MPC framework	Near-optimal economic performance
Mesbahi & Egerstedt	2010	Distributed Control	Graph-Theoretic	Multi-agent	Networked MEMS array control	Consensus stability analysis
Peng et al.	2021	Deep LSTM-MPC	LSTM + SQP on FPGA	MEMS Experimental	Real-time LSTM-MPC for MEMS	2.3 nm RMS tracking error
Li et al.	2023	ARE-GRU Predictive	ARE-GRU + Attention	MEMS Comb-Drive	Adaptive recalling GRU for MEMS	38% lower prediction RMSE
Wang et al.	2024	Hybrid ARE-LSTM-BO	Transformer + ARE-LSTM + PINN + BO	MEMS Two-Axis Scanner	Full integrated MEMS DL control	1.4–2.1 nm RMS tracking error

Analysis Based on Literature Review

The thirty studies reviewed demonstrate a clear evolution in adaptive predictive control for electrostatic MEMS nano positioning, beginning with foundational nonlinear dynamic analysis and progressing toward intelligent deep learning-based control frameworks. Early contributions by researchers such as Younis and Nayfeh established the importance of understanding pull-in instability and nonlinear electrostatic behavior, while Chu and Spanos introduced neural-network-based modeling approaches for microactuator systems. These studies collectively highlighted the limitations of conventional linear controllers and emphasized the need for adaptive, data-driven methodologies capable of handling highly nonlinear MEMS dynamics under strict computational constraints.

During the late 1990s and early 2010s, major theoretical developments laid the groundwork for ARE-RNN-based predictive control systems. The emergence of LSTM and GRU architectures enabled recurrent networks to capture long-term temporal dependencies in dynamic systems, significantly improving sequence prediction accuracy. At the same time, model predictive control theory provided stability and optimality guarantees for integrating neural surrogate models into real-time control environments. Optimization approaches such as Particle Swarm Optimization, Genetic Algorithms, and Bayesian Optimization further enhanced controller performance by enabling efficient hyperparameter tuning and adaptive learning within limited MEMS datasets.

More recent studies introduced attention mechanisms, transformer architectures, and physics-informed learning frameworks that substantially improved prediction reliability and adaptive control precision. Attention-based recurrent models selectively focus on critical temporal features, enhancing nano-positioning accuracy under disturbances and varying operating conditions. Hybrid ARE-RNN architectures integrating transformers, Bayesian optimization, and physics-informed regularization now represent the most advanced stage of development in MEMS predictive control research. Overall, the literature consistently demonstrates that combining physical system knowledge with deep learning architectures reduces prediction error, improves generalization, and enables more reliable real-time nano-positioning performance in electrostatic MEMS actuators.

Discussion

The reviewed literature demonstrates substantial progress in adaptive predictive control for electrostatic MEMS nano positioning, evolving from theoretical neural-network-based concepts to experimentally validated systems capable of nanometer-scale precision. ARE-RNN-based predictive control frameworks have emerged as highly effective solutions because they integrate nonlinear dynamics modeling, adaptive learning, and constrained optimization within a unified architecture. Unlike conventional controllers that depend heavily on accurate analytical modeling and manual parameter tuning, ARE-RNN approaches learn system behavior directly from data while adapting to uncertainties and environmental variations. The adaptive recalling mechanism further enhances performance by selectively retrieving important historical information, enabling improved prediction of long-term drift, transient disturbances, and multi-timescale dynamic behavior.

Despite these advances, several technical challenges continue to limit large-scale deployment in real-world MEMS applications. One major issue is the computational complexity of online optimization in model predictive control loops, particularly for embedded platforms with limited processing capability. Although FPGA-based implementations have achieved real-time operation, computational cost increases significantly with larger prediction horizons and higher-dimensional control problems. Researchers are therefore exploring explicit MPC approximations and offline optimization strategies to reduce online computation requirements and improve efficiency.

Another significant challenge involves model generalization across MEMS devices affected by fabrication-induced parameter variations such as residual stress, electrode geometry, and gap inconsistencies. Small structural differences can alter dynamic responses and reduce prediction accuracy. Transfer learning and domain adaptation techniques show promise in improving adaptability across devices. Additionally, sensor noise and thermal interference remain critical concerns at nanometer resolution. While noise-aware training and Kalman filtering partially mitigate these effects, robust uncertainty-aware learning frameworks and formal closed-loop stability guarantees remain important future research directions for safety-critical MEMS control systems.

Conclusion

This review systematically examined the integration of deep learning, optimization techniques, and model predictive control within Adaptive Recalling-Enhanced Recurrent Neural Network (ARE-RNN)-based predictive control for electrostatic MEMS nano positioning. By analyzing thirty representative studies, the review traced the evolution from early nonlinear dynamic modeling and neural system identification to advanced transformer-augmented ARE-LSTM architectures combined with physics-informed regularization and Bayesian optimization. The literature consistently demonstrates that combining adaptive memory mechanisms, temporal attention, physical system knowledge, and optimized learning strategies significantly improves prediction accuracy, robustness, and nano-scale positioning precision compared to conventional standalone methods.

The review also identified several promising future research directions. Physics-informed ARE-RNN models capable of embedding MEMS governing equations directly into neural architectures may further reduce training data requirements and improve generalization across varying operating conditions. Neuromorphic computing and spiking neural hardware offer opportunities for ultra-low-power real-time predictive control suitable for embedded MEMS systems. In addition, federated learning frameworks could support collaborative multi-device training while preserving proprietary calibration data privacy. Finally, formal stability certification through Lyapunov-based neural control design remains essential for deploying ARE-RNN predictive controllers in safety-critical medical, industrial, and aerospace nano-positioning applications.

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