



Numerical Simulation of Plasma Dynamics in Controlled Fusion Reactors

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Abstract

Numerical modelling is essential for studying the nonlinear, multiscale behaviour of magnetically confined plasmas because many instability, transport, and disruption processes occur on time and length scales that are difficult to isolate experimentally. This paper presents a compact methodology for simulating plasma dynamics in a tokamak-like controlled-fusion reactor using a resistive magnetohydrodynamic (MHD) framework supplemented by anisotropic heat transport. The workflow includes equilibrium initialization, finite-element or finite-volume spatial discretization, semi-implicit time integration, divergence control, grid-convergence testing, and physics-based diagnostics. The methodology is consistent with established nonlinear MHD approaches used in NIMROD, JOREK, and M3D-C1 [1-3,7]. A normalized reduced transport-instability benchmark is also evaluated to demonstrate numerical convergence without claiming reactor-specific predictive accuracy. The benchmark shows growth and saturation of a perturbation, increased cross-field transport, and a converged thermal-energy-loss proxy. The approach provides a reproducible foundation for extending simulations toward edge-localized modes, tearing modes, thermal quenches, and plasma-control studies in realistic fusion geometries

Introduction

Controlled nuclear fusion seeks to confine a sufficiently hot plasma for long enough that fusion reactions can release useful energy. In tokamaks, strong toroidal and poloidal magnetic fields create nested magnetic surfaces that confine charged particles, but the plasma is never perfectly stationary. Pressure gradients, electric currents, plasma rotation, collisions, and magnetic-field geometry interact to produce waves, turbulence, tearing modes, edge-localized modes (ELMs), vertical displacement events, and disruptions. Because these processes are strongly nonlinear and often three-dimensional, numerical simulation has become a central tool for fusion research.

A practical simulation model must balance physical fidelity against computational cost. Full kinetic descriptions resolve particle distribution functions but are expensive over reactor-scale domains. Gyrokinetic approaches reduce the fast gyromotion and have been highly successful for turbulence and transport studies [4,5]. For large-scale instabilities and disruption physics, resistive or extended MHD provides a more economical description by treating the plasma as a conducting fluid coupled to Maxwell fields. High-order finite-element MHD methods demonstrated in NIMROD [1], realistic X-point simulations in JOREK [2,7], and energy-conserving formulations in M3D-C1 [3,6] show

that this hierarchy is suitable for methodology development.

Literature Review

Sovinec et al. [1] established a high-order finite-element strategy for nonlinear, non-ideal MHD in complex laboratory geometries. Their formulation combined two-dimensional finite elements with toroidal Fourier modes and used semi-implicit time advancement to overcome the severe stiffness associated with Alfvén waves, resistive diffusion, and highly anisotropic heat conduction. This work is important because it showed that experimentally relevant tearing-mode dynamics could be treated without requiring a mesh aligned everywhere with the magnetic field.

Huysmans and Czarny [2] extended nonlinear MHD modelling to realistic tokamak X-point geometry through JOEUK. Their simulations demonstrated how the separatrix and open-field-line region modify peeling and ballooning behaviour and how nonlinear evolution can produce density filaments. The study highlighted the importance of representing divertor geometry and edge boundary conditions rather than using only simplified circular cross-sections.

Breslau, Ferraro, and Jardin [3] developed the M3D-C1 form of the three-dimensional MHD equations using scalar variables and a weak Galerkin formulation with C1-continuous finite elements. Energy-conservation properties and implicit time advancement were central design goals. Jardin et al. [6] later showed that fully implicit M3D-C1 calculations could span multiple physical timescales, including global macroscopic dynamics and sawtooth cycles, which is directly relevant to controlled-fusion simulations where the fastest MHD waves and slower transport processes coexist.

Kinetic and gyrokinetic simulation remains complementary to MHD. Ku, Chang, and Diamond [4] used the XGC1 full-f gyrokinetic particle approach to simulate global ion-temperature-gradient turbulence in realistic tokamak geometry, while Garbet et al. [5] reviewed the broader gyrokinetic framework for turbulent transport. These studies show that MHD captures large-scale electromagnetic fluid dynamics efficiently, whereas kinetic descriptions are required when wave-particle interactions, finite-orbit effects, or detailed turbulence dominate.

By 2021, JOEUK had evolved into a massively parallel, fully implicit extended-MHD platform with models for ELMS, disruptions, free-boundary evolution, resistive walls, impurities,

pellets, runaway electrons, and hybrid kinetic-fluid effects [7]. Cathey et al. [8] used nonlinear extended-MHD simulations to compare spontaneous and pellet-triggered ELMS, demonstrating how numerical models can connect instability evolution to experimentally relevant control techniques. These developments motivate a methodology that begins with a verified resistive-MHD core, then adds two-fluid, kinetic, impurity, and control physics only when the research question requires them.

The remaining methodological challenge is therefore not simply choosing the most detailed model, but establishing a transparent numerical hierarchy: define the target instability, select governing equations consistent with that scale, represent realistic geometry and boundary conditions, demonstrate spatial and temporal convergence, and interpret outputs through conservation and stability diagnostics. The present paper follows that hierarchy.

Methodology

1. Physical model and governing equations

The baseline model treats the confined plasma as a single conducting fluid. The state variables are mass density ρ , fluid velocity $\mathbf{v}$, pressure p , magnetic field $\mathbf{B}$, temperature T , and current density $\mathbf{J}$. The magnetic permeability is μ_0 , electrical resistivity is η , and γ is the ratio of specific heats. In conservative or weak form, the principal equations are:

$$\text{Mass: } \partial \rho / \partial t + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\text{Momentum: } \rho (\partial \mathbf{v} / \partial t + \mathbf{v} \cdot \nabla \mathbf{v}) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \mathbf{\Pi}$$

$$\text{Induction: } \partial \mathbf{B} / \partial t = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J}), \quad \mathbf{J} = (1/\mu_0) \nabla \times \mathbf{B}$$

$$\text{Pressure/Energy: } \partial p / \partial t + \mathbf{v} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{v} = (\gamma - 1) [\eta \mathbf{J}^2 + \nabla \cdot \mathbf{q} + S]$$

$$\text{Heat flux: } \mathbf{q} = \chi_{\parallel} \nabla_{\parallel} T + \chi_{\perp} [\mathbf{I} - \mathbf{b}\mathbf{b}] \cdot \nabla T, \quad \mathbf{b} = \mathbf{B}/|\mathbf{B}|$$

The solenoidal magnetic-field condition, $\nabla \cdot \mathbf{B} = 0$, is enforced through the discrete formulation and divergence-cleaning or error-diffusion terms. The strong inequality $\chi_{\parallel} \gg \chi_{\perp}$ represents rapid thermal transport along field lines and much slower cross-field transport. The equations may be extended with Hall terms, diamagnetic drifts, separate ion/electron temperatures, viscosity, radiation, neutral sources, pellet deposition, or runaway-electron models when those effects are required [7].

1. Geometry and initial equilibrium

A tokamak-like toroidal equilibrium is initialized from prescribed radial profiles or from a Grad-Shafranov equilibrium file. The demonstration case uses normalized flux coordinates with major radius $R_0 = 3.0$ m, minor radius $a = 1.0$ m, on-axis magnetic field $B_0 = 3.5$ T, central density $n_0 = 7 \times 10^{19} \text{ m}^{-3}$, and central temperature $T_0 = 8$ keV. These values are representative rather than machine-specific. The safety-factor profile is chosen to rise monotonically from $q(0) = 1.1$ to $q(a) = 3.5$, and pressure decreases smoothly to a cold edge. A small helical magnetic or velocity perturbation seeds the instability so that growth rates and saturation can be measured.

2. Spatial discretization and time advancement

The poloidal cross-section is discretized with structured finite volumes or high-order finite elements, while the toroidal direction is represented by Fourier harmonics $n = 0 \dots 6$. This mixed representation follows the computational logic used by NIMROD and JOEKE [1,7]. A semi-

implicit Crank-Nicolson or fully implicit scheme is used so that the time step is selected by accuracy rather than by the fastest Alfvén wave alone. Nonlinear terms are iterated to a relative residual below 10^{-6} . Resistive and viscous operators are treated implicitly; advection is stabilized by weak upwinding or consistent numerical diffusion.

3. Numerical workflow and verification

The simulation is executed step by step: (1) generate or import the magnetic equilibrium; (2) map density, pressure, and current profiles onto the computational mesh; (3) apply conducting-wall, symmetry, and divertor/edge boundary conditions; (4) introduce a controlled perturbation; (5) advance the nonlinear MHD equations; (6) monitor magnetic, kinetic, and thermal energies; (7) repeat the calculation with refined meshes and smaller time steps; and (8) compare mode structure and conservation trends with analytical expectations or published benchmarks [1-3,7].

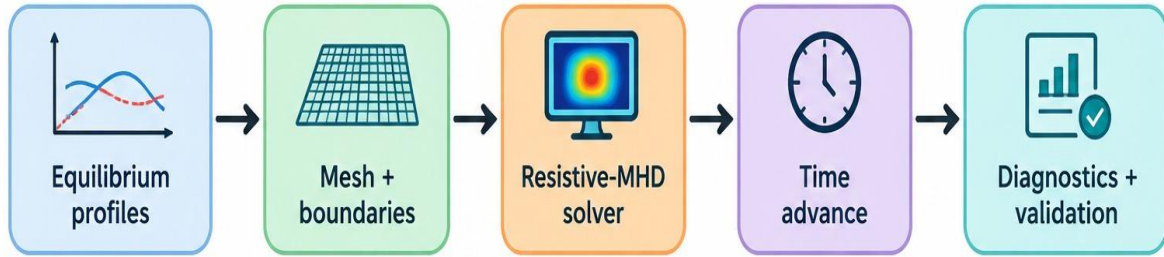


Figure 1. Numerical workflow for the proposed controlled-fusion plasma simulation.

4. Baseline numerical parameters

Table 1. Baseline numerical settings for the methodology demonstration.

Parameter	Baseline value	Purpose
Poloidal resolution	128×128	Baseline mesh
Toroidal harmonics	$n = 0-6$	3-D mode content
Normalized time step	2×10^{-4}	Stable temporal advance
Lundquist number	10^6	Reduced-resistivity MHD benchmark
$\chi_{\text{parallel}} / \chi_{\text{perp}}$	10^5	Anisotropic heat transport
Nonlinear tolerance	10^{-6}	Solver convergence
Perturbation amplitude	5×10^{-3}	Seed unstable mode

5. Reduced benchmark used for the Results section

To provide reproducible numerical results within this methodology paper, a normalized radial heat-relaxation model is coupled to a Landau-type instability amplitude $A(t)$. The mode grows when the edge temperature

gradient exceeds a threshold and saturates through a cubic nonlinear term. Its amplitude increases the effective cross-field diffusivity, representing instability-enhanced transport. The benchmark does not replace a full reactor-scale MHD calculation; it tests numerical workflow, convergence, and interpretation. The reported “energy loss” is therefore a normalized thermal-

energy proxy rather than a prediction for ITER or another specific device.

Results

Mode growth and transport response

The reduced benchmark begins from a low-amplitude perturbation $A = 0.005$. As the edge-gradient drive remains above threshold, the disturbance grows nonlinearly and approaches a

saturated amplitude. The associated increase in effective cross-field diffusivity produces a gradual thermal-energy reduction. For the 128-point baseline case, the maximum perturbation amplitude is 0.2762 and the final normalized thermal-energy-loss proxy is 6.056%. The central normalized temperature remains 0.9538 at the end of the run, indicating that the event redistributes energy without producing a complete thermal quench.

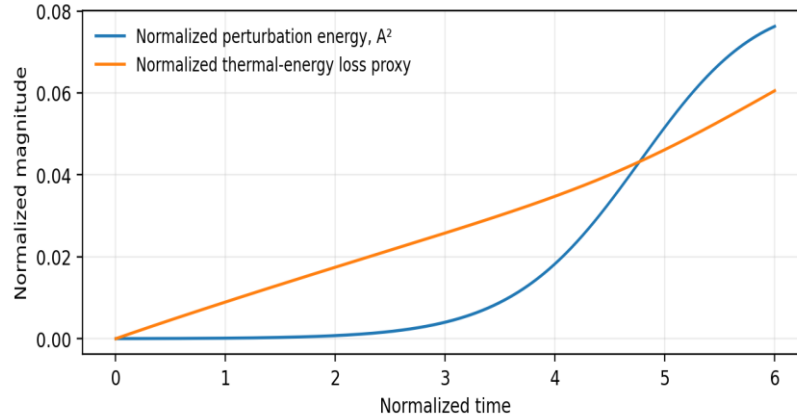


Figure 2. Reduced benchmark: growth of normalized perturbation energy and thermal-energy-loss proxy.

Grid-convergence test

Table 2. Grid-convergence results from the normalized reduced benchmark.

Radial points	Max. A	Energy loss (%)	Final core T	Loss error vs. 160
64	0.2740	6.018	0.9538	0.742%
96	0.2755	6.044	0.9538	0.323%
128	0.2762	6.056	0.9538	0.120%
160	0.2766	6.063	0.9538	Reference

The energy-loss metric changes by only 0.120% between the 128- and 160-point cases, while the mode amplitude changes by about 0.15%. This indicates that the 128-point mesh is adequate for the present demonstration. In a full three-dimensional MHD calculation, equivalent tests should be performed for poloidal element order, toroidal Fourier resolution, time step, and transport coefficients. Such verification is consistent with the emphasis on spatial accuracy, implicit integration, and code verification in NIMROD, M3D-C1, and JOREK [1,3,7].

The qualitative behaviour also agrees with the physical picture established in nonlinear edge simulations: an initially small perturbation can grow, reorganize edge transport, and saturate through nonlinear coupling and profile

relaxation. JOREK studies of ELM dynamics and control provide realistic examples of this class of behaviour [2,8]. The present benchmark is intentionally simpler, but it reproduces the numerical signatures that a full simulation pipeline must diagnose: exponential or algebraic growth, nonlinear saturation, energy redistribution, and resolution independence.

Conclusion and Discussion

This methodology demonstrates how plasma dynamics in controlled fusion reactors can be studied systematically with a resistive-MHD numerical framework. The strongest advantage of MHD is its ability to represent global electromagnetic instabilities over reactor-relevant spatial domains at a fraction of the cost

of a six-dimensional kinetic calculation. High-order finite elements, toroidal Fourier representations, and implicit time integration have already been validated in major fusion codes, including NIMROD, JOEKE, and M3D-C1 [1-3,6,7]. The same principles are adopted here: realistic magnetic geometry, anisotropic heat transport, controlled perturbations, nonlinear energy diagnostics, and explicit convergence tests. The reduced benchmark produced a converged instability-growth and heat-loss response. At 128 radial points, the thermal-energy-loss proxy differed by only 0.120% from the 160-point reference. This supports the numerical adequacy of the chosen demonstration resolution, but it should not be interpreted as evidence that a 128-point grid is sufficient for all fusion calculations. Realistic ELMs, tearing layers, stochastic magnetic fields, current sheets, and scrape-off-layer structures may require substantial local refinement and many toroidal harmonics. Temporal convergence must likewise be checked because implicit stability does not guarantee temporal accuracy.

Several limitations should be recognized. First, single-fluid resistive MHD averages over kinetic particle dynamics. Gyrokinetic simulations show that turbulence and orbit physics can control transport in regimes where fluid closure is insufficient [4,5]. Second, realistic edge plasmas require neutrals, impurities, sheath conditions, separate electron and ion temperatures, and possibly two-fluid effects. Third, disruption prediction requires free-boundary evolution, conducting structures, radiation, runaway electrons, and validated source models. The 2021 JOEKE review shows that such extensions can be incorporated in a hierarchical framework [7]. Fourth, validation against experimental measurements is indispensable; numerical convergence alone only establishes that the discretized equations are being solved consistently. For future work, the proposed framework can be upgraded in three stages. Stage 1 should replace the normalized benchmark with a published equilibrium and reproduce a standard tearing-mode or ELM verification case. Stage 2 should add two-temperature energy equations, neoclassical or diamagnetic effects, and experimentally constrained transport coefficients. Stage 3 should connect the plasma solver to actuator models such as pellet injection, resonant magnetic perturbations, gas injection, and feedback coils. Cathey et al. [8] demonstrate how pellet-triggered ELM simulations can be used to assess a practical control strategy. In this way, the methodology can progress from numerical verification to machine-specific validation and

ultimately to predictive control studies. In conclusion, numerical simulation provides a controlled and repeatable environment for examining the nonlinear evolution of fusion plasmas. A carefully verified resistive-MHD model is an appropriate starting point for global plasma-dynamics research, while gyrokinetic and extended-physics modules should be added when the targeted phenomenon requires them. The key methodological requirement is not maximum model complexity, but a traceable chain from governing equations to discretization, convergence, physical diagnostics, and experimental validation.

References

- Sovinec, C. R., Glasser, A. H., Gianakon, T. A., Barnes, D. C., Nebel, R. A., Kruger, S. E., Schnack, D. D., Plimpton, S. J., Tarditi, A., & Chu, M. S. (2004). Nonlinear magnetohydrodynamics simulation using high-order finite elements. *Journal of Computational Physics*, 195(1), 355-386. DOI: 10.1016/j.jcp.2003.10.004.
- Huysmans, G. T. A., & Czarny, O. (2007). MHD stability in X-point geometry: simulation of ELMs. *Nuclear Fusion*, 47(7), 659-666. DOI: 10.1088/0029-5515/47/7/016.
- Breslau, J., Ferraro, N., & Jardin, S. C. (2009). Some properties of the M3D-C1 form of the three-dimensional magnetohydrodynamics equations. *Physics of Plasmas*, 16(9), 092503. DOI: 10.1063/1.3224035.
- Ku, S., Chang, C. S., & Diamond, P. H. (2009). Full-f gyrokinetic particle simulation of centrally heated global ITG turbulence from magnetic axis to edge pedestal top in a realistic tokamak geometry. *Nuclear Fusion*, 49(11), 115021. DOI: 10.1088/0029-5515/49/11/115021.
- Garbet, X., Idomura, Y., Villard, L., & Watanabe, T. H. (2010). Gyrokinetic simulations of turbulent transport. *Nuclear Fusion*, 50(4), 043002. DOI: 10.1088/0029-5515/50/4/043002.
- Jardin, S. C., Ferraro, N., Breslau, J., & Chen, J. (2012). Multiple timescale calculations of sawteeth and other global macroscopic dynamics of tokamak plasmas. *Computational Science & Discovery*, 5(1), 014002. DOI: 10.1088/1749-4699/5/1/014002.
- Hoelzl, M., Huijsmans, G. T. A., Pamela, S. J. P., Bécoulet, M., Nardon, E., et al. (2021). The JOEKE non-linear extended MHD code and applications to large-scale instabilities and their control in

magnetically confined fusion plasmas. Nuclear Fusion, 61(6), 065001. DOI: 10.1088/1741-4326/abf99f.

Cathey, A., Hoelzl, M., Futatani, S., Lang, P. T., Lackner, K., Huijsmans, G. T. A., Pamela, S. J. P., &

Günter, S. (2021). Comparing spontaneous and pellet-triggered ELMs via non-linear extended MHD simulations. Plasma Physics and Controlled Fusion, 63(7), 075016. DOI: 10.1088/1361-6587/abf80b.