



Quantum Entanglement and Information Transfer in Multi-Qubit Computing Systems

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Abstract

Quantum entanglement is a central resource for quantum computation and state transfer, yet the usefulness of multipartite entanglement depends on how rapidly coherence and nonclassical correlations degrade as the register size and communication depth increase. This methodology paper develops a reproducible density-matrix simulation for studying entanglement and information transfer in multi-qubit systems. GHZ and W states containing 3, 4, 5, 6 and 8 qubits are generated and exposed to independent local depolarizing noise. State fidelity and bipartite negativity are used to quantify preservation of the prepared resource, while an entanglement-assisted multi-hop teleportation model measures the fidelity of transferred single-qubit information. The simulation shows a systematic decrease in state fidelity and negativity with increasing register size at a fixed local error probability. At a local depolarizing probability of 0.02, GHZ fidelity decreases from 0.941 for three qubits to 0.852 for eight qubits, whereas W-state fidelity decreases from 0.943 to 0.877. In a separate transfer experiment with an effective error probability of 0.10 per teleportation hop, average transfer fidelity decreases from 0.933 after one hop to 0.745 after five hops, while remaining above the classical benchmark of $2/3$. These results illustrate how multipartite entanglement quality; noise accumulation and path depth jointly determine the reliability of information transfer in multi-qubit computing architectures.

Introduction

Entanglement describes correlations in a composite quantum state that cannot be reduced to independent states of its subsystems. The conceptual roots of the subject appear in the Einstein-Podolsky-Rosen argument [1] and Bell's demonstration that local hidden-variable theories cannot reproduce all quantum predictions [2]. In quantum information science, this nonclassical correlation is treated as a resource rather than only a foundational puzzle. Bennett and Wiesner showed that a shared entangled pair can increase classical information capacity through dense coding [3], while Bennett et al. established quantum teleportation, in which

an unknown state is reconstructed at a remote qubit using a shared entangled pair and two classical bits [4]. Importantly, entanglement does not permit faster-than-light signalling because classical communication is still required for deterministic teleportation.

Modern quantum processors must extend these principles from isolated pairs to registers containing many interacting qubits. Multipartite states such as Greenberger-Horne-Zeilinger (GHZ), W and cluster states distribute correlations across multiple nodes and are therefore relevant to quantum networking, error-correcting architectures and

measurement-based computing. Experimental three-photon GHZ entanglement was demonstrated by Bouwmeester et al. [5]. Dür, Vidal and Cirac later showed that three-qubit GHZ and W states represent inequivalent classes of genuine tripartite entanglement [6], while Coffman, Kundu and Wootters formulated a monogamy relation that constrains how pairwise entanglement can be distributed within a three-qubit system [7]. These results motivate the use of more than one state family and more than one entanglement metric when evaluating multi-qubit performance.

The practical challenge is that entanglement is fragile. Gate errors, dephasing, readout errors and environmental coupling progressively reduce coherence, with the problem becoming more severe as circuit width and depth increase. The present study therefore asks two connected methodological questions: how does local noise change the quality of multipartite GHZ/W resources as the number of qubits increases, and how does repeat entanglement-assisted transfer change the fidelity of an unknown qubit as the number of network hops grows? A compact numerical methodology is developed so that both questions can be answered with the same density-matrix framework.

Literature Review

Entanglement theory provides several possible measures, each suited to a different task. The review by Horodecki et al. [8] synthesizes separability, distillation, Bell inequalities, entanglement witnesses and quantitative measures, emphasizing that no single measure captures every operational aspect of multipartite correlations. For the present methodology, state fidelity is used to quantify preservation of a target prepared state, while negativity is used as a computable indicator of bipartite entanglement across the partition 'first qubit versus the remaining register'. This combination is intentionally practical: fidelity reveals loss of the intended state even before all entanglement disappears, whereas negativity directly tests whether the chosen bipartition retains nonclassical correlations.

Experimental progress has steadily increased the scale at which multipartite entanglement can be generated and characterized. Monz et al. created and studied coherent 14-qubit entanglement in a trapped-ion system [9]. Friis et al. later generated and characterized entangled states in a fully controlled 20-qubit trapped-ion register, using scalable witnesses to detect genuine multipartite

entanglement among neighboring groups [10]. These studies demonstrate that large entangled registers are experimentally accessible, but also show that verification becomes more difficult as system size grows. Preskill's description of noisy intermediate-scale quantum (NISQ) hardware further emphasized that useful near-term processors are constrained by imperfect gates and finite coherent circuit depth [11].

The rapid expansion of programmable superconducting systems reinforced the importance of scalable verification. Arute et al. demonstrated a 53-qubit programmable superconducting processor for random-circuit sampling [12]. In 2020, Wei et al. introduced a scalable method based on multiple quantum coherences and reported multipartite GHZ entanglement extending across 18 qubits on a superconducting device [13]. These works support a methodology that explicitly studies the dependence of entanglement quality on register size rather than treating a Bell pair as a complete representation of processor-level behavior.

Information transfer provides the second operational test. Teleportation converts shared entanglement into an information-transfer channel [4]. By 2020, Huang, Huang and Li had proposed scalable networking teleportation protocols based on multi-qubit box and chain cluster states and implemented the box-cluster approach on a 14-qubit IBM device for networks containing multiple participants [14]. Their work highlights a key architectural point: as modular quantum systems grow, the number of intermediate operations and network nodes becomes an important performance variable. The present methodology abstracts this phenomenon into a controlled multi-hop transfer model so that transfer fidelity can be related directly to accumulated local noise.

Methodology

A simulation-based quantitative design was adopted. The calculations use complex state vectors and density matrices, allowing both ideal pure states and noisy mixed states to be represented without assuming a specific hardware technology. The workflow is summarized in Figure 1. Each stage produces numerical outputs that can be independently reproduced: state preparation, application of a noise channel, entanglement measurement, information-transfer simulation and comparison of fidelity against a classical teleportation benchmark.



Figure 1. Simulation workflow for evaluating multipartite entanglement and multi-hop information transfer.

Step 1 - State preparation. For each register size n in $\{3, 4, 5, 6, 8\}$, two target states are constructed. The n -qubit GHZ state represents global all-or-nothing coherence, whereas the W state distributes a single excitation among all qubits and retains a different entanglement structure [6].

$$|GHZ_n\rangle = (|0\rangle \otimes + |1\rangle \otimes) / \sqrt{2} \quad \text{and} \quad |W_n\rangle = (1/\sqrt{n}) \sum_k |0\dots 1_k \dots 0\rangle$$

Step 2 - Noise model. Independent Pauli depolarizing noise is applied to every qubit after ideal state preparation. For a single qubit, the channel is $E_p(\rho) = (1-p)\rho + (p/3)(X\rho X + Y\rho Y + Z\rho Z)$, where p is the local error probability. Applying the channel sequentially to each qubit provides a simple technology-independent model of accumulated local error. The principal register-size comparison uses $p = 0.02$. Additional checks at higher p are used to confirm monotonic degradation.

Step 3 - Entanglement metrics. Target-state fidelity is $F = \langle \psi | \rho | \psi \rangle$. A value of one indicates perfect preparation, while smaller values

indicate deviation from the ideal target. Bipartite negativity is calculated across the first-qubit/rest partition as $N = (||\rho^{TA}||_1 - 1)/2$, where T_A denotes partial transpose. For the ideal GHZ state this partition has $N = 0.5$; positive values after noise indicate that entanglement across the partition survives.

Step 4 - Information-transfer model. Quantum teleportation is modeled as a sequence of entanglement-assisted hops. The input ensemble contains six cardinal Bloch-sphere states: $|0\rangle$, $|1\rangle$, $|+\rangle$, $|-\rangle$, $|+i\rangle$ and $|-i\rangle$. Each hop is represented by an effective Pauli depolarizing map with $p = 0.10$, reflecting accumulated imperfections in entanglement preparation, Bell measurement, feed-forward and correction. The output fidelity for each input is calculated, and the six values are averaged. Path lengths from one to five hops are evaluated. The classical benchmark for average teleportation of an unknown qubit is $2/3$; therefore, average fidelity above 0.667 is interpreted as retaining quantum advantage under this idealized benchmark.

Table 1. Simulation design and evaluation parameters.

Parameter	Setting	Purpose
Register sizes	3, 4, 5, 6, 8 qubits	Tests scaling of multipartite states
State families	GHZ and W	Contrasts inequivalent multipartite structures
Register noise	Local depolarizing, $p = 0.02$	Fidelity and negativity comparison
Transfer inputs	6 cardinal single-qubit states	Unbiased average transfer test
Transfer depth	1–5 teleportation hops	Tests accumulated path error
Transfer noise	Effective depolarizing, $p = 0.10/\text{hop}$	Models' imperfect entanglement-assisted transfer
Primary metrics	Fidelity F ; negativity N	State preservation and entanglement survival

Step 5 - Analysis. Results are compared descriptively rather than through inferential statistics because the model is deterministic at fixed parameters. The principal trends are (i) change in GHZ/W fidelity with n , (ii) change in negativity with n and (iii) decay of average transfer fidelity with hop count. Reproducibility is obtained by fixing the state definitions, channel equation, qubit sizes and error probabilities in Table 1.

Step 6 - Numerical validation. Each generated density matrix is checked before metric calculation. The trace is required to remain unity within floating-point tolerance, Hermiticity is verified by comparing ρ with its conjugate transpose, and the minimum eigenvalue is inspected to ensure that the applied channel preserves positive semidefiniteness. The ideal $p = 0$ cases are used as internal controls: the prepared target state must return fidelity $F = 1$, and the first-qubit/rest negativity of the ideal

GHZ state must equal 0.5. These controls provide a direct test that the state construction, tensor ordering and partial-transpose implementation are consistent before noisy cases are interpreted.

Step 7 - Robustness analysis. To distinguish a genuine trend from a result tied to one arbitrary error setting, a limited sensitivity analysis is carried out for a six-qubit GHZ state at $p = 0.01, 0.02, 0.05$ and 0.10 . The same state preparation and measurement code is reused without retuning parameters. For information transfer, the hop count rather than the noise parameter is varied, so that accumulated depth is isolated as the independent variable. The six cardinal input states sample the three mutually unbiased Pauli axes and therefore test transfer of population and phase information rather than only computational-basis states.

Computational reproducibility. The complete simulation can be reproduced using standard complex linear algebra without access to proprietary hardware. A state vector of dimension 2^n is first converted to a density matrix $\rho = |\psi\rangle\langle\psi|$. For each qubit, full-register Pauli operators are formed by tensoring X, Y or Z at the target position with identity matrices on all other positions, after which the local depolarizing map is applied. Partial transpose is

implemented by reshaping the density matrix into subsystem indices and exchanging the bra and ket indices of the selected qubit. Eigenvalues of the partially transposed matrix give negativity, while direct projection onto the target state gives fidelity. Reporting these explicit operations makes the methodology transferable to MATLAB, Python/NumPy, Mathematica or a gate-level simulator.

Results

Table 2 reports the calculated multipartite-state metrics at $p = 0.02$ per qubit. Both target-state fidelity and negativity decrease as n increases, demonstrating cumulative sensitivity to local noise. GHZ fidelity falls from 0.9413 at $n = 3$ to 0.8519 at $n = 8$. W-state fidelity remains slightly higher over the tested sizes, decreasing from 0.9435 to 0.8772. The negativity trend differs: GHZ states retain greater first-qubit/rest negativity than W states for $n \geq 3$ under this partition, with GHZ negativity decreasing from 0.4545 to 0.3967 and W negativity decreasing from 0.4277 to 0.2892. The result is consistent with the fact that fidelity and partition-specific entanglement quantify different properties of the noisy state.

Table 2. Simulated fidelity and bipartite negativity under local depolarizing noise ($p = 0.02$ per qubit).

Qubits (n)	State	Fidelity (F)	Negativity (N)
3	GHZ	0.9413	0.4545
3	W	0.9435	0.4277
4	GHZ	0.9226	0.4424
4	W	0.9288	0.3890
5	GHZ	0.9043	0.4305
5	W	0.9151	0.3565
6	GHZ	0.8865	0.4189
6	W	0.9021	0.3298
8	GHZ	0.8519	0.3967
8	W	0.8772	0.2892

The information-transfer experiment shows a comparable accumulation effect with network depth. At $p = 0.10$ per hop, average fidelity is 0.9333 after one hop, 0.8756 after two, 0.8255 after three, 0.7821 after four and 0.7445 after five. Thus, the fifth-hop value is about 20.2% lower than the one-hop value, although it

remains above the $2/3$ classical benchmark within the assumptions of the model. This result demonstrates why modular quantum architectures require not only high-quality entanglement at individual links but also short routing paths, high-fidelity Bell operations and effective error mitigation.

Table 3. Average simulated quantum-state transfer fidelity versus teleportation path depth ($p = 0.10$ per hop).

Teleportation hops	Average fidelity	Relative to 2/3 benchmark
1	0.9333	Above
2	0.8756	Above
3	0.8255	Above

4	0.7821	Above
5	0.7445	Above

A sensitivity check also confirms monotonic degradation for a six-qubit GHZ state: its fidelity is approximately 0.942 at $p = 0.01$, 0.886 at $p = 0.02$, 0.738 at $p = 0.05$ and 0.542 at $p = 0.10$. Consequently, the combined evidence supports the methodological hypothesis that both register width and transfer depth amplify the impact of local noise. The study does not assert that GHZ or W states are universally superior; rather, it shows that the preferred resource depends on the operational metric and the partition or protocol under consideration.

Conclusion and Discussion

This paper presented a compact numerical methodology for investigating quantum entanglement and information transfer in multi-qubit computing systems using literature and concepts available through 2020. The framework combines two multipartite state families, a transparent local depolarizing channel, target-state fidelity, bipartite negativity and a multi-hop teleportation test. Simulation results show that noise accumulation with increasing qubit count reduces both state fidelity and entanglement, while repeated transfer operations progressively reduce the fidelity of a transported unknown qubit. At the selected parameters, the transfer fidelity remains above the $2/3$ classical benchmark through five hops, but the declining margin illustrates the challenge of extending entanglement-assisted communication across deeper quantum networks. The results should be interpreted as methodological and illustrative rather than as measurements from a particular processor. Real devices have nonuniform gate errors, amplitude damping, dephasing, crosstalk, connectivity limits and measurement errors that are not captured by a single depolarizing parameter. Furthermore, negativity across one bipartition does not fully characterize genuine n -partite entanglement. Future implementations can therefore extend the framework by replacing the depolarizing map with calibrated device noise, evaluating multiple bipartitions or entanglement witnesses, and comparing chain, cluster and error-corrected teleportation protocols. Within these limitations, the proposed approach provides a reproducible baseline for linking entanglement quality to reliable information transfer in multi-qubit architectures. A further methodological implication concerns the difference between state quality and communication quality. High multipartite-state fidelity is desirable, but it is not by itself a

guarantee of high end-to-end transfer fidelity. A network protocol consumes entanglement through Bell measurements, classical feed-forward and conditional corrections, so every additional link introduces opportunities for error even when each individual resource state is acceptable. Conversely, a state whose global fidelity is modest may still preserve useful entanglement across a specific partition. For this reason, the framework intentionally reports both a state-overlap metric and a correlation metric before evaluating the operational transfer task. This separation helps prevent the common mistake of treating one scalar fidelity as a complete description of an n -qubit resource. For practical device studies, the same workflow can be extended in three directions. First, hardware-calibrated amplitude-damping and phase-damping channels can replace the symmetric Pauli model, allowing T1/T2 processes to be represented explicitly. Second, measured gate and readout error rates can be assigned to individual qubits so that connectivity-dependent paths can be compared rather than assuming identical links. Third, multipartite verification can be strengthened by adding entanglement witnesses or evaluating negativity across several bipartitions. Such extensions would make it possible to distinguish whether transfer degradation is dominated by state preparation, entangling gates, measurement or path length, which is essential when the methodology is applied to a real NISQ processor.

References

- Einstein, B. Podolsky, and N. Rosen, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" *Physical Review*, vol. 47, pp. 777–780, 1935. DOI: 10.1103/PhysRev.47.777.
- J. S. Bell, "On the Einstein Podolsky Rosen Paradox," *Physics Physique Fizika*, vol. 1, pp. 195–200, 1964. DOI: 10.1103/PhysicsPhysiqueFizika.1.195.
- H. Bennett and S. J. Wiesner, "Communication via One- and Two-Particle Operators on Einstein-Podolsky-Rosen States," *Physical Review Letters*, vol. 69, pp. 2881–2884, 1992. DOI: 10.1103/PhysRevLett.69.2881.
- H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an Unknown Quantum State via Dual Classical and

Einstein-Podolsky-Rosen Channels,” *Physical Review Letters*, vol. 70, pp. 1895–1899, 1993. DOI: 10.1103/PhysRevLett.70.1895.

Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, and A. Zeilinger, “Observation of Three-Photon Greenberger-Horne-Zeilinger Entanglement,” *Physical Review Letters*, vol. 82, pp. 1345–1349, 1999. DOI: 10.1103/PhysRevLett.82.1345.

W. Dür, G. Vidal, and J. I. Cirac, “Three Qubits Can Be Entangled in Two Inequivalent Ways,” *Physical Review A*, vol. 62, 062314, 2000. DOI: 10.1103/PhysRevA.62.062314.

V. Coffman, J. Kundu, and W. K. Wootters, “Distributed Entanglement,” *Physical Review A*, vol. 61, 052306, 2000. DOI: 10.1103/PhysRevA.61.052306.

R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, “Quantum Entanglement,” *Reviews of*

Modern Physics, vol. 81, pp. 865–942, 2009. DOI: 10.1103/RevModPhys.81.865.

T. Monz et al., “14-Qubit Entanglement: Creation and Coherence,” *Physical Review Letters*, vol. 106, 130506, 2011. DOI: 10.1103/PhysRevLett.106.130506.

N. Friis et al., “Observation of Entangled States of a Fully Controlled 20-Qubit System,” *Physical Review X*, vol. 8, 021012, 2018. DOI: 10.1103/PhysRevX.8.021012.

J. Preskill, “Quantum Computing in the NISQ Era and Beyond,” *Quantum*, vol. 2, p. 79, 2018. DOI: 10.22331/q-2018-08-06-79.

Arute et al., “Quantum Supremacy Using a Programmable Superconducting Processor,” *Nature*, vol. 574, pp. 505–510, 2019. DOI: 10.1038/s41586-019-1666-5.