



Theoretical Analysis of Dark Matter Distribution in Galactic Halo Structures

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Peer Review Information	Abstract
<i>Submission: 19 April 2020</i> <i>Revision: 05 May 2020</i> <i>Acceptance: 25 May 2020</i> Keywords <i>Dark matter galactic halos, NFW profile, Burkert profile, Lambda-CDM</i>	Dark matter dominates the gravitational mass budget of galactic halos and therefore controls the formation, rotation, and long-term dynamical stability of galaxies. This methodology paper presents a theoretical framework for analysing dark-matter distributions through analytical halo density models. The Navarro-Frenk-White (NFW), Burkert, and Einasto profiles are normalized to a common virial mass and compared using radial density, enclosed mass, logarithmic density slope, and circular velocity. A fiducial Milky-Way-mass halo of $M_{200} = 10^{12} M_{\text{sun}}$ is adopted under a flat Lambda-CDM cosmology consistent with Planck-era parameters. The resulting calculations show that cuspy and cored prescriptions can agree well at intermediate and outer halo radii while predicting substantially different densities and circular velocities in the inner few kiloparsecs. This behaviour illustrates why the central structure of dwarf and spiral galaxies remains a sensitive test of the combined effects of collisionless cold dark matter, baryonic feedback, and halo assembly history. The framework is deliberately analytical and reproducible, making it suitable as a baseline methodology for later comparison with observed rotation curves or cosmological simulation outputs.

Introduction

Galactic rotation curves, gravitational lensing, satellite dynamics, and cosmological structure formation all imply that galaxies are embedded in extended halos whose mass greatly exceeds the directly visible stellar and gaseous components. In the Lambda-CDM paradigm, these halos arise through hierarchical gravitational clustering of predominantly collisionless cold dark matter. High-resolution N-body work by Navarro, Frenk, and White established that simulated equilibrium halos can be represented by a nearly universal two-parameter density law with an inner cusp and an outer r^{-3} decline (Navarro et al., 1997). Large simulation campaigns later confirmed that halo structure is closely related to assembly history, concentration, and substructure (Springel et al., 2008; Dutton & Maccio, 2014).

The main difficulty arises at small radii. Rotation-curve studies of dark-matter-dominated systems have often favoured shallower central distributions than the r^{-1} -like cusp expected for an NFW halo. Early analyses formulated this disagreement as a central-density or core-cusp problem (Burkert, 1995; Kravtsov et al., 1998), and later reviews showed that improved velocity fields reduced some observational systematics without eliminating the underlying modelling tension (de Blok, 2010). Modern hydrodynamic simulations have demonstrated that repeated gas inflow and outflow can modify the central gravitational potential and transform cusps into cores over selected ranges of stellar-to-halo mass ratio (Di Cintio et al., 2014; Read et al., 2016). Consequently, a theoretical analysis should compare alternative density profiles under the

same halo-scale normalization rather than treat one profile as universally valid.

The objective of this study is therefore to construct a transparent, reproducible methodology for comparing dark-matter halo structures. The analysis uses only theoretical profiles and derived quantities; it does not claim new observational measurements. Specifically, it asks how the assumed inner density law changes the predicted radial density and circular velocity of an otherwise identical halo, and which quantities provide the clearest discrimination between cuspy and cored models.

Literature Review

Navarro et al. (1997) provided the most widely used baseline for collisionless halo structure. Their NFW profile links the characteristic density and scale radius to halo formation and yields a logarithmic inner slope approaching -1. Burkert (1995) proposed a phenomenological cored form for dwarf galaxies, replacing the central divergence with an approximately constant-density region. Kravtsov et al. (1998) further emphasized that comparisons between simulations and observed rotation curves must account for scaling, scatter, and uncertainty in the inferred baryonic contribution.

The Aquarius Project greatly improved the numerical resolution of Milky-Way-sized dark matter halos and showed that their subhalo populations and internal structures are broadly consistent with collisionless Lambda-CDM expectations, while also exposing the importance of halo-to-halo variance (Springel et al., 2008). The concentration-mass relation provides an

additional structural constraint: Dutton and Maccio (2014) showed that both NFW and Einasto descriptions can be calibrated in a Planck-era cosmology, with concentration varying systematically with halo mass and redshift.

At galaxy scales, baryons complicate the collisionless picture. Di Cintio et al. (2014) found a systematic dependence of the best-fitting dark-matter profile on stellar-to-halo mass ratio, implying that feedback can alter the inner slope most efficiently over a restricted mass range. Read et al. (2016) demonstrated in high-resolution dwarf-galaxy simulations that sustained star formation can generate cores comparable to the stellar half-mass radius. Bullock and Boylan-Kolchin (2017) reviewed these and related small-scale Lambda-CDM challenges and stressed that baryonic physics, observational inference, and possible dark-matter microphysics must be separated carefully when testing the paradigm.

By 2020, FIRE-2 calculations had extended this picture across a wide range of galaxy masses. Lazar et al. (2020) introduced a core-Einasto form and found that feedback-driven core formation is strongest near a characteristic stellar-to-halo mass ratio, whereas very low-mass galaxies remain close to cuspy dark-matter-only expectations. In parallel, the final Planck cosmological-parameter analysis continued to support a spatially flat six-parameter Lambda-CDM background, providing a consistent cosmological normalization for theoretical halo calculations (Planck Collaboration, 2020).

Table 1. Representative halo models and their theoretical interpretation

Model / source	Inner behavior	Outer behavior	Main parameters	Interpretive use
NFW; Navarro et al. (1997)	Cuspy, $\rho \propto r^{-1}$	$\rho \propto r^{-3}$	ρ_s , r_s or M_{200} , c_{200}	Collisionless Lambda-CDM baseline
Burkert; Burkert (1995)	Finite-density core	$\rho \propto r^{-3}$	ρ_0 , r_0	Empirical cored comparison
Einasto; Dutton & Maccio (2014)	Continuously varying slope	Steepens smoothly	ρ_2 , r_2 , α	High-resolution halo fitting
Feedback-modified; Di Cintio et al. (2014); Lazar et al. (2020)	Cusp or core depends on galaxy formation	Approaches halo-scale form	Profile shape plus stellar/halo ratio	Baryonic modification of central halo

The literature therefore motivates a comparative methodology: the virial mass should be held fixed while the central profile shape is varied, and the analysis should examine both density and dynamical consequences. This design isolates the structural effect of the assumed halo law from changes in the total halo mass.

Methodology

The methodology is organized as a sequence of analytical steps so that the calculations can be reproduced with a spreadsheet, numerical script, or symbolic package. No observational fitting is performed in the present paper.

Step 1 - Adopt a cosmological and halo definition

A flat Lambda-CDM background is adopted using $H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_m = 0.315$, consistent with the Planck 2020 cosmological-parameter results (Planck Collaboration, 2020). The halo boundary is defined at a mean interior density of 200 times the present critical density. For a chosen virial mass M_{200} , the critical density and virial radius are calculated from:

$$\rho_{\text{crit}} = 3H_0^2 / (8\pi G), \quad r_{200} = [3M_{200} / (4\pi \cdot 200\rho_{\text{crit}})]^{1/3}.$$

For the fiducial $M_{200} = 1.0 \times 10^{12} M_{\text{sun}}$ halo, the calculation gives $r_{200} \approx 211.6 \text{ kpc}$. This halo mass is representative of a Milky-Way-scale system and is used only to demonstrate the method.

Step 2 - Define the NFW cuspy reference profile

The NFW density law is used as the collisionless reference because of its long-standing role in cosmological halo studies (Navarro et al., 1997; Springel et al., 2008). With $x = r/r_s$,

$$\rho_{\text{NFW}}(r) = \rho_s / [x(1+x)^2].$$

The concentration is $c_{200} = r_{200}/r_s$. A fiducial $c_{200} = 10$ is adopted, giving $r_s \approx 21.16 \text{ kpc}$. The normalization ρ_s is solved by requiring $M(<r_{200}) = M_{200}$, where $M(r) = 4\pi\rho_s r_s^3 [\ln(1+x) - x/(1+x)]$. The adopted concentration lies within the broad range expected for galaxy-scale halos in concentration-mass studies (Dutton & Maccio, 2014).

Step 3 - Define the Burkert cored comparison

The Burkert model is selected as a simple cored alternative motivated by dwarf-galaxy mass distributions (Burkert, 1995; de Blok, 2010). With $y = r/r_0$,

$$\rho_B(r) = \rho_0 / [(1+y)(1+y^2)].$$

A core radius $r_0 = 8 \text{ kpc}$ is used for the illustrative calculation. The central density ρ_0 is not chosen arbitrarily; it is solved so that the Burkert halo has exactly the same M_{200} and r_{200} as the NFW halo. This common virial-mass normalization ensures that differences at small radius are caused primarily by profile shape rather than by total mass.

Step 4 - Add an Einasto smooth-slope model

An Einasto profile is included because modern simulations often show continuously varying logarithmic slopes rather than a single fixed inner power law (Dutton & Maccio, 2014). The adopted expression is

$$\rho_E(r) = \rho_{-2} \exp\{-(2/\alpha)[(r/r_{-2})^\alpha - 1]\}.$$

For the demonstration $\alpha = 0.18$ and $r_{-2} = r_s$ are used. The density normalization ρ_{-2} is obtained numerically from the same condition $M(<r_{200}) = M_{200}$. The Einasto curve therefore acts as an intermediate high-resolution simulation-oriented benchmark rather than an additional free-mass model.

Step 5 - Compute dynamical quantities

Each model is sampled over $0.5 \text{ kpc} \leq r \leq r_{200}$ on a logarithmic radial grid. The enclosed mass is calculated by analytic integration for NFW and Burkert and by numerical quadrature for Einasto. The model circular velocity is

$$V_c(r) = \sqrt{[G M(<r) / r]}.$$

The circular-velocity curve is a useful bridge between the density model and observable galaxy kinematics. The logarithmic density slope, $d \ln \rho / d \ln r$, is also evaluated conceptually: values near -1 indicate a cusp, whereas values approaching 0 toward the centre indicate a core. Because baryons can modify the central halo through repeated potential fluctuations, any comparison with real galaxies should ultimately include stellar and gas mass components (Di Cintio et al., 2014; Read et al., 2016; Lazar et al., 2020).

Step 6 - Apply common normalization and internal checks

Three checks are imposed. First, each profile must recover $M_{200} = 10^{12} M_{\text{sun}}$ at r_{200} within numerical round-off. Second, the density must remain positive and monotonically declining across the sampled radial range. Third, the derived circular velocity must be real and continuous. These checks are important because profile comparisons can otherwise be distorted by inconsistent virial definitions or independent normalization choices.

Step 7 - Compare central and halo-scale behavior

The analysis focuses on representative radii of 1, 5, 8, 20, 50, 100, and 200 kpc. Inner points highlight the cusp-core difference; intermediate points probe the region near the scale radius and expected rotation-curve maximum; outer points test whether the different profiles converge after being forced to share the same virial mass. The outputs are density and circular velocity for each model. No statistical significance is quoted because the study contains no observational error distribution.

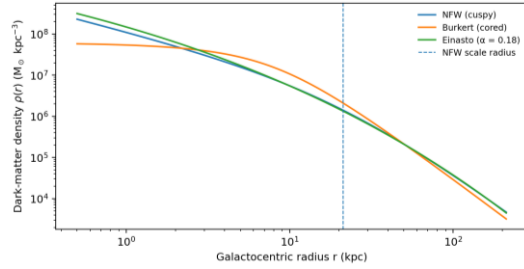


Figure 1. Theoretical NFW, Burkert, and Einasto density profiles normalized to the same $M_{200} = 10^{12} M_{\text{sun}}$ halo. The figure is generated from the analytical methodology used in this study.

Figure 1 illustrates the principal methodological expectation: the largest separation occurs in the inner halo. At tens to hundreds of kiloparsecs, normalization to the same M_{200} causes the profiles to approach one another, while the central density retains strong sensitivity to the adopted functional form.

Results

The calculations produce $r_{200} = 211.6$ kpc and $r_s = 21.2$ kpc for the fiducial halo. All three models are normalized to $10^{12} M_{\text{sun}}$ at r_{200} . The resulting values should be interpreted as model predictions under the specified assumptions rather than as observations of the Milky Way.

Table 2. Illustrative analytical density and circular-velocity results

r (kpc)	ρ_{NFW}	ρ_{Burkert}	ρ_{Einasto}	$V_{c,\text{NFW}}$	$V_{c,\text{B}}$	$V_{c,\text{E}}$
1	1.09e+08	5.38e+07	1.48e+08	55.1	31.7	64.1
5	1.56e+07	2.72e+07	1.71e+07	110.1	125.0	120.0
8	7.86e+06	1.54e+07	8.02e+06	129.4	164.5	136.5
20	1.58e+06	2.42e+06	1.51e+06	161.0	211.7	161.7
50	2.11e+05	2.12e+05	2.10e+05	171.7	202.7	170.2
100	3.64e+04	2.90e+04	3.74e+04	163.0	176.0	162.8
200	5.46e+03	3.78e+03	5.30e+03	144.4	145.1	144.4

Density units: $M_{\text{sun}} \text{ kpc}^{-3}$; circular velocity units: km s^{-1} .

At 1 kpc, the NFW model predicts $\rho \approx 1.09 \times 10^8 M_{\text{sun}} \text{ kpc}^{-3}$, while the chosen Burkert core gives $\approx 5.38 \times 10^7 M_{\text{sun}} \text{ kpc}^{-3}$. The difference is a direct consequence of the NFW cusp and the finite Burkert central density. The Einasto model, with $\alpha = 0.18$ and the adopted normalization, is also centrally dense but its slope varies continuously with radius rather than approaching one fixed power law.

The circular-velocity response is not identical to the local-density response because V_c depends on the entire enclosed mass. At 1 kpc, V_c is about 55.1 km s^{-1} for NFW and 31.7 km s^{-1} for

Burkert. By 5-20 kpc, the cored model has accumulated enough mass that its V_c can exceed the NFW value for the particular $r_0 = 8$ kpc normalization adopted here. At 200 kpc, all three circular velocities are nearly the same because the total enclosed masses have been forced to converge at r_{200} .

This result demonstrates an important inference principle. A model that is clearly different in central density can still reproduce similar outer-halo dynamics. Consequently, discrimination among halo profiles requires data at sufficiently small radii and an accurate decomposition of stars, gas, and dark matter. The same logic underlies the observational core-cusp literature summarized by de Blok (2010) and the

simulation-based feedback studies of Di Cintio et al. (2014), Read et al. (2016), and Lazar et al. (2020).

1. Sensitivity of the theoretical result

The numerical values vary if M_{200} , concentration, core radius, or Einasto shape parameter is changed. Increasing c_{200} raises the inner NFW density at fixed M_{200} , while increasing the Burkert core radius redistributes mass over a broader central region. Thus, the table should not be interpreted as a unique prediction for every galaxy. Its purpose is to show how a controlled, common-mass comparison isolates structural sensitivity.

Conclusion and Discussion

This paper developed a theoretical methodology for analysing dark-matter distributions in galactic halo structures using three established analytical density profiles. The approach begins with a cosmological virial definition, fixes M_{200} and r_{200} , assigns profile-specific structural parameters, normalizes every model to the same virial mass, and derives density, enclosed mass, logarithmic slope, and circular velocity over a common radial grid. This sequence is straightforward to reproduce and avoids a frequent source of confusion in halo comparisons: models with different central shapes should not also be allowed to carry unrelated total masses unless that difference is intentionally being tested. For the fiducial $10^{12} M_{\odot}$ example, the strongest model dependence appears in the inner few kiloparsecs. The NFW law retains a cusp, whereas the Burkert law produces a finite-density core. The Einasto model provides a smoothly changing-slope alternative that resembles the flexible fitting forms used for high-resolution simulated halos. At large radii, the models become dynamically similar because they share the same virial normalization. The result is fully consistent with the long-standing theoretical distinction between collisionless cusps (Navarro et al., 1997; Springel et al., 2008) and cored profiles inferred or generated in some galaxy-scale contexts (Burkert, 1995; de Blok, 2010; Read et al., 2016). The broader interpretation is that central halo structure is not determined by dark-matter-only gravity in isolation once galaxy formation becomes important. Di Cintio et al. (2014) and Lazar et al. (2020) show that the response of the inner halo depends systematically on the efficiency of star formation relative to halo mass. Repeated, rapid changes in the baryonic gravitational potential can transfer energy to dark matter and flatten the central density, while in other mass regimes

baryonic contraction or weak feedback can preserve or enhance central concentration. Bullock and Boylan-Kolchin (2017) therefore caution that small-scale Lambda-CDM tests must distinguish astrophysical modification from genuinely non-standard dark-matter physics.

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