



Numerical Simulation of Plasma Confinement in Magnetic Fusion Reactors

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Abstract

Magnetic-confinement fusion requires plasma temperatures high enough for thermonuclear reactions while minimizing radial energy and particle losses. Because direct experimental exploration of every operating regime is expensive and limited by machine availability, numerical simulation is a central tool for predicting confinement behavior and testing transport hypotheses. This methodology paper develops a reproducible reduced numerical framework for studying plasma confinement in a tokamak-like magnetic fusion reactor. The approach combines a one-dimensional radial energy-transport equation, a parameterized effective diffusivity representing turbulence-driven transport, finite-volume discretization, and systematic sweeps of magnetic-field strength and flow-shear level. The numerical procedure is informed by the development of gyrokinetic turbulence simulation, zonal-flow physics, and modern transport modeling reported in the fusion literature up to 2020. Illustrative calculations show that lowering the effective turbulent diffusivity produces more peaked temperature profiles and a larger energy-confinement proxy. In the representative sweep, the normalized confinement proxy increases from about 0.86 for a low-field, unsheared case to about 3.37 for the highest-field, strongest-shear case. These results are not presented as an ITER discharge prediction; rather, they demonstrate a transparent methodology that can be extended to experimental equilibria, kinetic closures, electromagnetic fluctuations, and high-fidelity gyrokinetic solvers. The framework is suitable for methodological studies in which numerical verification, parameter sensitivity, and physically interpretable confinement trends are required.

Introduction

A magnetic fusion reactor attempts to confine a hot, ionized plasma long enough that fusion heating can compete with energy losses. In a tokamak, strong toroidal and poloidal magnetic fields guide charged particles along helical field lines and reduce direct cross-field motion. The remaining transport, however, is often dominated by collective instabilities and turbulence rather than by simple binary-collision diffusion. This makes energy confinement a multiscale problem: particle gyromotion occurs

on very small spatial and temporal scales, while equilibrium profiles and machine geometry vary on much larger scales. Numerical models are therefore required to connect microscopic turbulence to macroscopic quantities such as temperature profiles, heat flux, and confinement time.

The scientific foundation for computational confinement studies was strengthened by large-scale gyrokinetic simulations showing that self-generated zonal flows can suppress turbulent

transport (Lin et al., 1998). Subsequent benchmark comparisons demonstrated that gyrokinetic, gyrofluid, and transport models can agree on many linear features yet differ substantially in predicted nonlinear transport, emphasizing the need for numerical convergence and model verification (Dimits et al., 2000). Electron-scale simulations further showed that electron-temperature-gradient turbulence can produce experimentally relevant thermal losses (Dorland et al., 2000). These findings motivate a methodology in which the transport closure, discretization, and sensitivity of the predicted confinement are stated explicitly rather than treated as hidden numerical details.

Literature Review

The numerical description of confined plasma turbulence developed rapidly from particle-based and reduced-fluid calculations toward sophisticated Eulerian and global gyrokinetic solvers. Lin et al. (1998) used massively parallel gyrokinetic simulations to establish the importance of zonal flows in reducing turbulence-driven transport. Their work demonstrated that nonlinear self-organization is not a secondary detail: turbulence can generate sheared flows that, in turn, regulate the turbulence itself. This feedback remains central to modern explanations of improved confinement and provides the physical basis for including a flow-shear control parameter in reduced transport studies.

Dimits et al. (2000) compared several tokamak transport models and turbulence simulations for common benchmark conditions. Although linear growth rates and frequencies were often similar, nonlinear transport levels could differ substantially, with numerical resolution, system size, and particle statistics contributing to the variation. The so-called Dimits-shift behavior also highlighted that nonlinear stabilization can extend the effective turbulence threshold beyond the linear instability threshold. For methodology design, this literature establishes two requirements: model assumptions must be stated clearly, and results should be accompanied by grid or parameter-sensitivity checks.

At smaller spatial scales, Dorland et al. (2000) reported toroidal gyrokinetic simulations of electron-temperature-gradient turbulence and found that electron-scale structures can generate significant heat transport. The implication is that a complete reactor calculation may require multiple turbulence scales or justified closures that represent unresolved scales. Candy and

Waltz (2003) subsequently presented a Eulerian gyrokinetic-Maxwell solver capable of nonlinear electromagnetic calculations with drift-kinetic electrons and radial profile variation. Their verification against analytic theory and other numerical models illustrates the importance of benchmark testing before a code is used for predictive confinement analysis.

Global geometry also matters. Idomura et al. (2003) developed a global gyrokinetic toroidal particle code for ion-temperature-gradient turbulence in reactor-relevant tokamak parameters. Unlike local flux-tube calculations, global simulations can retain profile variation and nonlocal radial coupling, which are important when turbulence spreads or when the equilibrium evolves. Garbet et al. (2010) reviewed the gyrokinetic framework, numerical techniques, and turbulence-transport physics in fusion devices, consolidating the view that first-principles gyrokinetics is the reference theory for microturbulent transport while reduced models remain valuable for integrated and rapid studies.

Flow shear provides another route to confinement improvement. Highcock et al. (2010) numerically showed that sheared rotation can shift the nonlinear turbulence threshold and produce a transport bifurcation to states with larger temperature and velocity gradients. Barnes et al. (2011) derived and numerically tested scaling arguments for ion-temperature-gradient turbulence, while Abel et al. (2013) formulated a multiscale gyrokinetic framework linking fluctuations, equilibrium transport, and energy flows in rotating tokamak plasmas. Together, these studies justify reduced methodology parameters that represent magnetic-field strength, gradient drive, and shear-mediated turbulence suppression.

By 2020, confinement modeling increasingly combined high-fidelity simulation with faster reduced or data-driven tools. Van de Plassche et al. (2020) demonstrated a neural-network surrogate for quasilinear tokamak transport that reproduced the parent model much faster, illustrating the value of reduced computational models for scenario exploration. Hakim et al. (2020) developed continuum electromagnetic gyrokinetic algorithms for turbulence in open-field-line and scrape-off-layer configurations, extending robust kinetic simulation toward reactor-edge conditions. Wang et al. (2020) used global flux-driven gyrokinetic simulations to analyze intermittent and non-diffusive heat-transport structures. These studies show that a modern methodology should preserve a clear path from simple, interpretable transport

equations to progressively richer gyrokinetic physics.

High-fidelity gyrokinetic codes provide the most detailed representation of turbulence but are computationally demanding and require specialized equilibrium, collision, and boundary-condition treatments. For a methodology paper focused on transparent numerical experimentation, there is value in a reduced model that preserves the core causal chain—transport coefficient $\rightarrow$ radial heat loss $\rightarrow$ temperature profile $\rightarrow$ confinement metric—while allowing controlled sensitivity tests. The present study therefore uses a reduced radial transport equation as a methodological bridge rather than claiming to replace gyrokinetic simulation.

Methodology

1. Physical and mathematical model

The computational domain represents the minor-radius direction of an axisymmetric tokamak plasma. The normalized radial coordinate is $x = r/a$, where r is the local minor radius and a is the plasma minor radius. Density is treated as fixed in the demonstration so that the numerical experiment isolates thermal confinement. The dependent variable is a normalized ion-temperature field $T(x,t)$. A centrally peaked heating profile represents auxiliary and fusion-like power deposition, while a linear sink represents aggregated volumetric losses not resolved explicitly.

$$\partial T / \partial t = (1/x) \partial / \partial x [x \chi_{\text{eff}} \partial T / \partial x] + S(x) - \nu_L T$$

Here, χ_{eff} is the effective radial thermal diffusivity, $S(x) = S_0 \exp[-(x/0.33)^2]$ is a normalized central heat source, and ν_L is a loss coefficient. At the magnetic axis, symmetry requires $\partial T / \partial x = 0$. At the plasma edge, T is fixed at a small normalized value to represent coupling to a colder edge region. This equation is intentionally simpler than a gyrokinetic system;

it is used because each term can be interpreted, independently varied, and numerically verified.

2. Parameterized confinement closure

To reproduce the qualitative effect that stronger magnetic field and stronger $E \times B$ -like flow shear can reduce turbulent transport, the study uses a positive-definite parameterization for χ_{eff} . The expression is a controlled sensitivity model, not a universal empirical scaling law. A finite transport floor prevents χ_{eff} from approaching zero and mimics residual neoclassical or unresolved transport.

$$\chi_{\text{eff}} = \chi_{\text{floor}} + \chi_0 (B_{\text{ref}}/B)^2 / [1 + 2.5 \gamma E^2]$$

In the reference calculation, $\chi_0 = 0.18$, $\chi_{\text{floor}} = 0.025$, $B_{\text{ref}} = 5$ T, and the dimensionless shear parameter γE is varied from 0 to 0.8. This form makes the assumed physics transparent: increasing B or γE decreases the modeled turbulent diffusivity. The motivation for the shear dependence is qualitative consistency with zonal-flow and externally sheared-flow suppression reported in gyrokinetic studies (Lin et al., 1998; Highcock et al., 2010), while the need for transport-model sensitivity follows from the model comparisons of Dimits et al. (2000) and the broader review by Garbet et al. (2010).

3. Numerical discretization

The radial interval $0 \leq x \leq 1$ is divided into 220 uniformly spaced nodes. The diffusion operator is written in conservative cylindrical form and discretized by a second-order finite-volume approximation. The magnetic-axis condition is imposed with the symmetry limit of the cylindrical Laplacian, while the outer boundary is imposed directly as a Dirichlet condition. For the steady calculations reported here, $\partial T / \partial t$ is set to zero and the resulting tridiagonal linear system is solved directly. The residual vector $R = A T - b$ is evaluated after each solve, and solutions are repeated on course and finer grids to confirm that the reported confinement trends are not artifacts of radial resolution.

Table 1. Reference simulation inputs and numerical settings

Quantity	Symbol	Reference value / range	Purpose
Normalized radius	$x = r/a$	0–1	Radial computational domain
Radial nodes	N	220	Finite-volume resolution
Magnetic field	B	3, 5, 7 T	Field-strength sensitivity
Flow-shear parameter	γE	0.0, 0.4, 0.8	Turbulence-suppression sensitivity
Transport floor	χ_{floor}	0.025	Residual transport
Loss coefficient	ν_L	0.08	Aggregated unresolved losses

Edge temperature	T _{edge}	0.05	Dirichlet condition	boundary
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Step-by-step simulation procedure

Step 1 – Define the confinement problem. Select the tokamak-like radial domain, the magnetic-field cases, the flow-shear cases, the heat-source profile, and the loss model.

Step 2 – Construct the transport coefficient. Calculate χ_{eff} for each B and γE pair using the declared reduced closure. Record χ_{eff} before solving so the sensitivity mechanism is auditable.

Step 3 – Generate the numerical mesh. Create the uniform radial nodes and face locations required by the conservative finite-volume form of the cylindrical diffusion operator.

Step 4 – Assemble the linear system. For all interior nodes, compute inward and outward

diffusive flux coefficients. Apply symmetry at $x = 0$ and the fixed edge temperature at $x = 1$.

Step 5 – Solve and verify. Solve $A T = b$, calculate the algebraic residual, and repeat selected cases at different N. A method is accepted only when the trend and reported metrics change negligibly under refinement.

Step 6 – Compute confinement indicators. Calculate the central temperature, maximum radial gradient, stored-energy proxy $W = \int T \, dx$, input-power proxy $P = \int S \, dx$, and $\tau E^* = W/P$.

Step 7 – Perform the parameter sweep. Compare all B- γE cases and identify how reduced χ_{eff} alters profile peaking and the normalized confinement proxy.

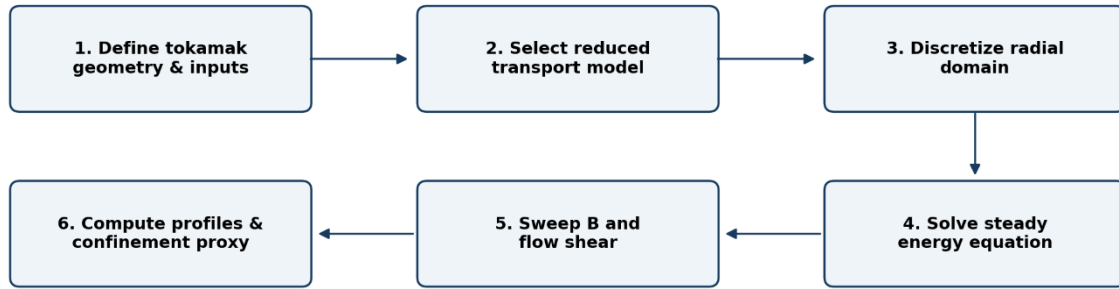


Figure 1. Step-by-step workflow used for the reduced plasma-confinement simulation.

4. Verification and analysis plan

Three forms of verification are used. First, the algebraic residual is checked to confirm that the discrete equations are solved accurately. Second, grid sensitivity is assessed by comparing the reference 220-node calculation with 110- and 440-node solutions for representative cases. Third, qualitative trend verification is performed: if χ_{eff} is deliberately reduced while all other inputs remain unchanged, the model should produce a steeper core-to-edge temperature profile and a larger stored-energy-to-power ratio. This monotonic trend is a mathematical consequence of the declared reduced model and therefore serves as a useful internal check. Physical validation against a

specific tokamak discharge would be a separate activity requiring measured equilibrium, density, temperature, rotation, heating, impurities, and edge boundary conditions.

Results

The reduced model was solved for seven representative magnetic-field and shear combinations. Table 2 summarizes the effective diffusivity and the resulting confinement indicators. All values except B are normalized or model-specific; they should therefore be interpreted as comparative numerical outputs rather than direct reactor performance predictions.

Table 2. Illustrative numerical results from the parameter sweep

B (T)	γE	χ_{eff}	T(0)	$\tau E^* = W/P$	Max $ dT/dx $
3	0.0	0.525	0.190	0.864	0.196
3	0.8	0.217	0.379	1.400	0.463

5	0.0	0.205	0.397	1.453	0.490
5	0.4	0.154	0.506	1.757	0.646
5	0.8	0.094	0.763	2.464	1.021
7	0.4	0.091	0.789	2.533	1.058
7	0.8	0.060	1.103	3.369	1.530

For the low-field unsheared case ($B = 3$ T, $\gamma E = 0$), χ_{eff} is 0.525 and the normalized central temperature is only 0.190. The corresponding confinement proxy is 0.864. When the shear parameter is increased to 0.8 at the same field, χ_{eff} falls to 0.217, the central temperature rises to 0.379, and the confinement proxy increases to 1.400. This comparison isolates the modeled effect of shear-mediated turbulence reduction.

At the reference field $B = 5$ T, the same trend is stronger because the baseline diffusivity is lower. The unsheared case gives $\tau E^* = 1.453$, whereas $\gamma E = 0.8$ gives $\tau E^* = 2.464$. At $B = 7$ T and $\gamma E = 0.8$, χ_{eff} reaches 0.060 and τE^* reaches 3.369, approximately 3.9 times the low-field unsheared value. The temperature gradient also increases because improved confinement permits a larger core-to-edge temperature difference for the same normalized source and edge condition.

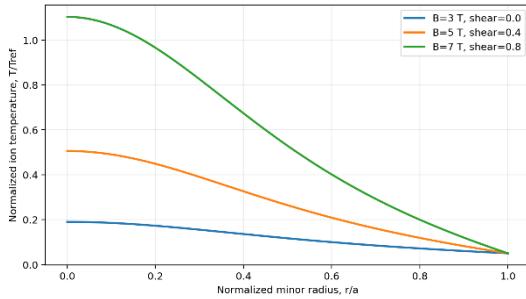


Figure 2. Illustrative steady radial temperature profiles for low-, reference-, and high-confinement parameter sets.

Figure 2 shows that the numerical mechanism is physically interpretable within the limits of the reduced model. A larger χ_{eff} flattens the temperature profile because heat diffuses outward efficiently. A smaller χ_{eff} allows central heating to produce a more strongly peaked profile. This is consistent with the general confinement picture established by gyrokinetic turbulence studies, where reduced turbulent transport is associated with stronger profile gradients and improved energy retention (Garbet et al., 2010; Highcock et al., 2010). It also aligns qualitatively with global simulations showing that transport can be regulated by

coherent structures, zonal flows, and non-diffusive events rather than a single fixed diffusivity (Wang et al., 2020).

Discussion

The principal result of the methodology is not a specific numerical value but a reproducible chain between assumptions and confinement indicators. In the reduced equation, magnetic-field strength and flow shear affect confinement only through χ_{eff} . Consequently, the model makes an explicit hypothesis: cases with stronger field or stronger shear experience lower effective cross-field thermal transport. The parameter sweep then quantifies how strongly the temperature profile and stored-energy proxy respond to that hypothesis. This structure is useful for methodology papers because it separates the physical closure from the numerical algorithm and from the derived metrics.

The results should not be interpreted as a substitute for first-principles gyrokinetics. Gyrokinetic simulations retain velocity-space dynamics, finite-Larmor-radius effects, self-consistent fluctuating electromagnetic fields, multiple particle species, and instability physics that a scalar diffusivity cannot reproduce. The importance of such effects is documented by electron-scale turbulence calculations (Dorland et al., 2000), Eulerian gyrokinetic-Maxwell modeling (Candy & Waltz, 2003), global toroidal calculations (Idomura et al., 2003), and multiscale formulations (Abel et al., 2013). In addition, the scrape-off layer and pedestal require special boundary physics, strong fluctuations, and often electromagnetic full-f methods, as emphasized by Hakim et al. (2020).

Nevertheless, reduced transport simulation remains valuable when the research objective is parameter exploration, integrated scenario modeling, or rapid uncertainty analysis. The 2020 development of fast surrogate transport models based on large quasilinear databases demonstrates this computational need (van de Plassche et al., 2020). A practical multilevel

workflow can therefore begin with the present model, replace χ_{eff} with a quasilinear or neural-network closure, and finally validate selected operating points with a nonlinear gyrokinetic code. This hierarchy reduces computational cost while preserving a route to higher physical fidelity.

Conclusion

This paper presented a step-by-step numerical methodology for analyzing plasma confinement in a tokamak-like magnetic fusion reactor. The framework uses a one-dimensional cylindrical radial energy equation, a declared turbulence-transport parameterization, conservative finite-volume discretization, direct steady-state solution, grid verification, and systematic magnetic-field and flow-shear sweeps. The illustrative numerical results show that reducing the effective thermal diffusivity produces higher central temperature, steeper profiles, and a larger normalized energy-confinement proxy. Across the selected cases, τE^* increased from about 0.86 to 3.37 as the assumed transport was reduced. The study therefore demonstrates how confinement trends can be investigated transparently without misrepresenting reduced-model outputs as experimental reactor predictions. Future work should replace the scalar closure with physics-based turbulent fluxes, couple density and electron/ion energy equations, include rotation and magnetic geometry from equilibrium reconstruction, and compare the numerical predictions against measured tokamak discharges. For reactor-scale studies, selected cases should be benchmarked against verified gyrokinetic or extended-MHD solvers. Such extensions would retain the present methodology—clear assumptions, numerical verification, sensitivity analysis, and traceable confinement metrics—while increasing predictive capability.

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