



Application of Finite Element Methods in Electromagnetic Field Simulations

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Peer Review Information

Submission: 30 April 2015

Revision: 09 May 2015

Acceptance: 26 May 2015

Keywords

Android APK, Malware Detection, Machine Learning, Random Forests, Software Security

Abstract

Finite element methods (FEM) provide a flexible numerical framework for electromagnetic field simulation when geometry, material interfaces, and boundary conditions make analytical solutions impractical. This methodology paper presents a compact workflow for electrostatic and time-harmonic electromagnetic analysis based on Maxwell's equations, weak formulations, mesh discretization, and appropriate finite-element spaces. Particular emphasis is placed on curl-conforming edge elements, which preserve tangential field continuity and reduce nonphysical spurious modes in vector wave problems. A reproducible axisymmetric coaxial electrostatic benchmark is used to demonstrate the validation stage of the workflow. Linear FEM solutions are compared with the closed-form logarithmic potential distribution while the radial mesh is refined from 10 to 80 elements. The relative electric-field error decreases from 0.388% to 0.0062%, with an observed convergence rate close to second order for the selected error measure. The study shows that reliable electromagnetic FEM depends not only on solving a matrix system but also on choosing a physically compatible formulation, controlling mesh quality, applying correct boundary conditions, and verifying the numerical solution against analytical or reference data. The proposed procedure can be extended to waveguides, eddy-current devices, antennas, scattering problems, and heterogeneous electromagnetic media.

Introduction

Electromagnetic design problems are governed by Maxwell's equations, but closed-form field solutions are available only for relatively simple geometries and idealized material distributions. Engineering systems such as transformers, electric machines, waveguides, microwave components, sensors, shielding structures, and scattering objects commonly contain curved boundaries, multiple media, local field concentrations, and open-domain conditions. FEM is well suited to these problems because the physical domain can be subdivided into small elements, the governing differential

equations can be recast in weak form, and material or geometric complexity can be represented locally [12].

For electromagnetic vector fields, the finite-element space must reflect the mathematical structure of Maxwell's equations. Nédélec introduced mixed edge-based elements that became a foundation for H(curl)-conforming discretizations [1]. Subsequent work established vector and tangential elements for practical electromagnetic computation and clarified how tangential continuity across element boundaries can suppress spurious solutions that may

appear when conventional nodal interpolation is applied directly to vector wave equations [2,3]. Monk provided convergence analysis for time-harmonic Maxwell problems [4], while later higher-order and adaptive developments improved accuracy and computational efficiency [5–8].

The objective of this paper is to present a step-by-step FEM methodology that is technically sound yet sufficiently compact for reproducible electromagnetic simulation. The method covers problem definition, equation selection, weak formulation, discretization, assembly, solution, post-processing, and verification. A simple coaxial benchmark is then solved numerically so that mesh-convergence behavior can be quantified without relying on unpublished experimental data.

Literature Review

The development of electromagnetic FEM has been driven by the need to preserve the differential and continuity properties of Maxwell fields. Nédélec's mixed finite elements in three dimensions established basis functions associated with edges rather than only with vertices [1]. Cendes later formulated vector finite elements for electromagnetic field computation, emphasizing consistent vector transformations and high-order constructions [2]. Lee, Sun, and Cendes developed tangential vector finite elements and showed how edge-based degrees of freedom provide an effective route to higher-order vector approximations [3]. These contributions formed the practical basis for modern curl-conforming FEM.

Theoretical analysis accompanied these computational developments. Monk analyzed a finite-element approximation for time-harmonic Maxwell equations and provided error estimates

in curl-conforming spaces [4]. Graglia, Wilton, and Peterson presented systematic higher-order interpolatory vector bases, demonstrating faster convergence when polynomial order is increased [5]. Demkowicz and Vardapetyan extended this direction through hp-adaptive formulations for electromagnetic absorption and scattering, combining local mesh refinement with variable approximation order [6]. Hiptmair subsequently unified much of the mathematical framework through discrete differential forms and explained why finite-element spaces compatible with the de Rham sequence are central to stable computational electromagnetism [7].

Research from 2012–2015 increasingly emphasized adaptivity, solver robustness, and implementation efficiency. Zhong et al. established convergence and optimality of adaptive edge-element procedures for time-harmonic Maxwell equations [8]. Ciarlet, Wu, and Zou studied edge-element methods with strong convergence properties for Gauss' laws [9]. Chung et al. demonstrated three-dimensional edge-FEM modeling of controlled-source electromagnetic surveys with complex conductivity contrasts [10]. Anjam and Valdman showed that vectorized assembly can substantially accelerate edge-element matrix construction in MATLAB [11]. Kuczmanski summarized practical FEM steps for static magnetic and eddy-current fields [12], while Hu et al. developed an effective preconditioner for an edge-FEM perfectly matched layer system for electromagnetic scattering [13]. Together, these works support a methodology in which field-compatible elements, adaptive resolution, efficient assembly, and solver conditioning are treated as parts of one simulation process rather than independent choices.

Table 1. Representative development of FEM for electromagnetic simulation (references through 2015).

Study	Main contribution	FEM feature	Relevance
Nédélec (1980) [1]	Mixed finite elements in 3-D	Edge/curl-conforming basis	Foundation for Maxwell FEM
Monk (1992) [4]	Error analysis for time-harmonic Maxwell equations	Curl-conforming discretization	Convergence theory
Graglia et al. (1997) [5]	Higher-order vector bases	p-refinement	Improved accuracy
Demkowicz &	hp-adaptive scattering	h- and p-adaptivity	Local error control

Vardapetyan (1998) [6]	formulation		
Zhong et al. (2012) [8]	Adaptive edge FEM convergence	A posteriori refinement	Optimal adaptive workflow
Anjam & Valdman (2015) [11]	Fast matrix assembly	Vectorized edge-element code	Efficient implementation
Hu et al. (2015) [13]	Preconditioned PML Maxwell system	Edge FEM + PML	Open-domain scattering

Methodology

The proposed methodology uses a model-discretize-solve-verify sequence (Figure 1). The same structure is applicable to electrostatic,

magnetostatic, eddy-current, and full-wave problems, although the field variable and finite-element space change with the governing equations.

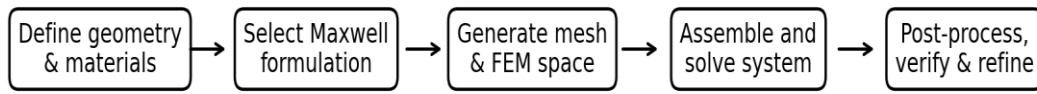


Figure 1. Step-by-step finite-element workflow for electromagnetic field simulation

1. Governing equations and problem classification

The first step is to classify the electromagnetic regime. Electrostatic models use $\nabla \cdot (\epsilon \nabla V) = -\rho$ and $E = -\nabla V$. Magnetostatic models may be written in terms of a magnetic vector potential A . For sinusoidal steady-state full-wave analysis with $\exp(j\omega t)$ dependence, an electric-field form can be written as $\nabla \times (\mu^{-1} \nabla \times E) - \omega^2 \epsilon E = -j\omega J$. In conducting media, complex material terms can include conductivity σ . The computational domain Ω , material tensors ϵ and μ , source J , and boundary conditions must be defined before meshing. Perfect electric conductor boundaries impose $n \times E = 0$; symmetry, impedance, periodic, or absorbing/PML boundaries are selected according to the physical problem [9,13].

2. Weak formulation and finite-element space

FEM is obtained by multiplying the governing equation by an admissible test function, integrating over Ω , and transferring derivatives by integration by parts. For electrostatics this produces a scalar H^1 problem. For vector Maxwell equations, the natural space is $H(\text{curl})$, and Nédélec edge elements are preferred because the tangential field component remains continuous across neighboring elements [1,7]. In matrix form, a frequency-domain problem has the generic structure $(K - \omega^2 M + j\omega C)e = f$, where K is the curl-curl stiffness matrix, M is the permittivity/mass matrix, C represents

conductive or damping contributions, e is the vector of field degrees of freedom, and f is the excitation vector.

3. Mesh generation, assembly, and numerical solution

The geometry is partitioned into triangles or quadrilaterals in two dimensions and tetrahedra or hexahedra in three dimensions. Mesh density should increase near corners, narrow gaps, material interfaces, skin-depth regions, and sources. For wave problems, the element size must also resolve the local wavelength. Element matrices are evaluated by numerical quadrature and assembled into sparse global matrices. Vectorized assembly and sparse storage are important for large systems [11]. Linear systems may be solved by sparse direct methods or by iterative methods with suitable preconditioners; open-region scattering simulations may combine edge elements with absorbing layers and specialized block preconditioners [10,13].

4. Verification benchmark used in this paper

To verify the numerical workflow, an axisymmetric coaxial electrostatic model is used. The inner conductor radius is $a=10$ mm, the outer conductor radius is $b=50$ mm, $V(a)=100$ V, and $V(b)=0$ V. The dielectric region is homogeneous. The exact potential is $V(r)=V_0 \ln(b/r)/\ln(b/a)$, and the exact radial electric-field magnitude is $E(r)=V_0/[r \ln(b/a)]$. The one-dimensional radial weak form is $\int_a^b$

$r(dV/dr)(dw/dr)dr=0$. The interval is discretized using linear finite elements. For an element $[r_1, r_2]$ of length h , the local stiffness contribution is proportional to $(r_1+r_2)/(2h)[[1,-1],[-1,1]]$. Dirichlet values are

imposed at the two conductor boundaries, the reduced sparse system is solved, and numerical potential and field values are compared with the analytical expressions.

Table 2. Parameters and verification criteria for the coaxial FEM benchmark.

Parameter	Value	Purpose
Inner radius, a	10 mm	Inner conductor
Outer radius, b	50 mm	Outer conductor
Boundary potentials	100 V; 0 V	Dirichlet conditions
Mesh levels	10, 20, 40, 80 elements	Convergence study
Primary metrics	Maximum $ V_h - V $; relative field error	Verification

5. Step-by-step simulation protocol

Step 1 - Define the objective and outputs. Specify whether the target quantity is electric potential, electric-field intensity, magnetic flux density, current density, impedance, resonant frequency, scattering parameter, or absorbed power. The required output determines the governing field variable and the accuracy metric.

Step 2 - Construct and simplify the geometry. Create the physical domain and retain geometric details that materially affect the field. Very small features may be removed only after checking that they do not alter current paths, material interfaces, local capacitance, or field enhancement.

Step 3 - Assign materials and sources. Enter relative permittivity, permeability, electrical conductivity, impressed currents, voltages, charges, or incident fields. Frequency-dependent or anisotropic properties should be represented explicitly when they are significant.

Step 4 - Select boundary and truncation conditions. Use Dirichlet or Neumann conditions for scalar-potential problems, perfect-electric or perfect-magnetic conditions for wave problems where appropriate, and impedance, radiation, or PML treatments for open domains. Boundary placement should be tested for sensitivity.

Step 5 - Select the finite-element family and order. Use nodal Lagrange elements for scalar electrostatic potentials. For vector Maxwell equations, use curl-conforming Nédélec/edge elements so that tangential continuity is represented correctly. Higher-order bases can be considered when smooth solutions permit p -refinement [5].

Step 6 - Generate an initial mesh and assemble the system. Resolve interfaces, corners, skin-depth regions, source neighborhoods, and wavelengths. Evaluate element integrals, assemble sparse global matrices, apply

constraints, and solve the reduced system. Efficient assembly and preconditioning become increasingly important as the number of degrees of freedom grows [11,13].

Step 7 - Post-process physically meaningful quantities. Recover E , H , B , D , current density, stored energy, losses, forces, or port quantities from the solved degrees of freedom. Derived fields should be interpreted with the polynomial order of the primary variable in mind because numerical differentiation can reduce smoothness.

Step 8 - Verify and document convergence. Repeat the solution on refined meshes or with higher polynomial order and compare a selected scalar error measure. Check conservation or Gauss-law behavior where relevant [9]. Record mesh size, element order, solver tolerance, material data, and boundary placement so the simulation can be reproduced.

6. Quality-control criteria

A simulation is accepted only after three checks. First, the algebraic solver must reach a residual tolerance appropriate to the discretization error; excessive iterative error can mask mesh convergence. Second, at least two successive refinements should show a stable trend in the chosen output. Third, the model should be compared with an analytical benchmark, measured result, or trusted reference case whenever available. For open-domain full-wave models, the outer-boundary or PML thickness must also be varied to demonstrate that truncation does not control the answer. These checks distinguish numerical verification from physical validation and reduce the risk of reporting a visually plausible but mesh-dependent field map.

Results

The numerical potential follows the analytical logarithmic profile closely even on the moderate

20-element mesh (Figure 2). Because the potential is interpolated linearly inside each element, the derived electric field is piecewise constant; nevertheless, field accuracy improves rapidly as the radial mesh is refined. The

benchmark therefore illustrates the verification stage that should precede application to geometrically complicated electromagnetic models.

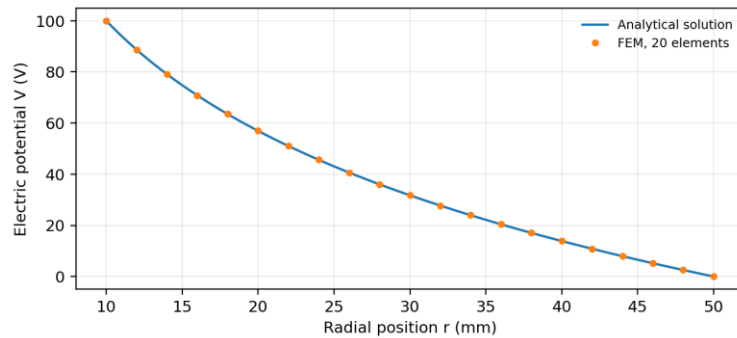


Figure 2. FEM and analytical electric-potential distributions for the coaxial benchmark.

Table 3. Mesh-convergence results for the electrostatic verification problem.

Elements	Nodes	Max. potential error (V)	Relative E-field error (%)	Observed order
10	11	0.1359	0.3884	
20	21	0.0349	0.0988	1.98
40	41	0.0088	0.0248	1.99
80	81	0.0022	0.0062	2.00

With 10 elements, the maximum nodal potential error is 0.1359 V and the relative electric-field error is 0.3884%. Increasing the mesh to 20, 40, and 80 elements reduces the field error to 0.0988%, 0.0248%, and 0.0062%, respectively. The estimated rate based on successive field-error reductions approaches 2.0. This behavior confirms stable mesh convergence for the benchmark and demonstrates how a quantitative error criterion can be used instead of visual agreement alone. In a full Maxwell simulation, the same principle is applied using field norms, energy error, resonant-frequency error, scattering parameters, or an a posteriori estimator; adaptive edge-element strategies can then concentrate degrees of freedom where the estimated error is largest [8].

Conclusion and Discussion

The methodology demonstrates that FEM is particularly valuable in electromagnetics because it combines local geometric flexibility with variational formulations that can respect Maxwell field continuity. The literature shows a clear progression from foundational edge elements [1–3], through convergence theory and higher-order bases [4–7], to adaptive methods, efficient implementation, and robust open-domain solvers [8–13]. For vector electromagnetic waves, selecting $H(\text{curl})$ -

conforming edge elements is a physical and mathematical requirement rather than a purely numerical preference. Mesh refinement should be linked to wavelength, skin depth, interfaces, singular regions, and a measurable error indicator. The coaxial benchmark provides a transparent validation example: refinement from 10 to 80 elements reduces the relative field error by more than sixty times. Although this electrostatic model is simpler than a three-dimensional scattering or eddy-current problem, it captures the essential verification logic. A credible FEM study should first reproduce a known solution, then perform mesh or polynomial-order convergence checks, and only afterward apply the model to a configuration for which no exact solution is available. For time-harmonic open-domain problems, additional validation should include boundary/PML sensitivity and solver-residual checks. For high-frequency or strongly heterogeneous problems, hp-adaptivity and suitable preconditioning can reduce computational cost while preserving accuracy [6,8,13]. In conclusion, the proposed step-by-step procedure—define the physical model, select the Maxwell formulation, derive the weak form, choose compatible finite elements, generate and refine the mesh, assemble and solve the sparse system, post-process fields, and

verify against analytical or reference data—provides a reliable basis for electromagnetic field simulation. Future extensions may incorporate nonlinear magnetic materials,

coupled thermal-electromagnetic analysis, moving conductors, and three-dimensional adaptive edge-element models.

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