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Investigation of Some Magnetized Cosmological Model with cosmological constant Λ-Term in Bimetric Theory of Gravitation

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Abstract

In this paper, some magnetized cosmological model with cosmological constant Λ in Rosen's bimetric theory of gravitation is investigated by using the techniques of Letelier and Stachel. The nature of the model is discussed in presence and in absence of the magnetic field as well as in the presence and in the absence of cosmological constant. Our model exists and it never goes to vacuum model for cosmological constant is not equal to zero and for finite value of cosmic time and it approaches to vacuum model when the cosmic time tends to minus infinity, in the presence of magnetic field. Further in the absence of magnetic field, model exists if cosmological constant is negative and n lies between zero and half and our model filled with dark matter if cosmological constant is positive and n greater than half.

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INTRODUCTION

The Rosen's bimetric theory(1977) is the theory of gravitation based on two metrics. One is the fundamental metric tensor g_{ij} describes the gravitational potential and the second metric γ_{ij} refers to the flat space–time and describes the inertial forces associated with the acceleration of the frame of reference. The metric tensors g_{ij} determine the Riemannian geometry of the curved space time which plays the same role as given in Einstein's general relativity and it interacts with matter. The background metric γ_{ij} refers to the geometry of the empty universe (no matter but gravitation is there) and describes the inertial forces. The metric tensor γ_{ij} has no direct physical significance but appears in the field equations. Therefore it interacts with g_{ij} but not directly with matter. One can regard γ_{ij} as giving the geometry that would exists if there were no matter. In the absence of matter one would have $g_{ij} = \gamma_{ij}$.

Thus at every point of space–time, there are two metrics

$$ds^2 = g_{ij}dx^i dx^j \quad , \qquad (1)$$

$$d\eta^2 = \gamma_{ij}dx^i dx^j \quad . \qquad (2)$$

The field equations of Rosen's (1974) bimetric theory of gravitation are

$$N_i^j - \frac{1}{2}N\delta_i^j + \Lambda\ g_i^j = -8\pi\, kT_i^j \quad , \qquad (3)$$

where $N = N_i^i$, $k = \sqrt{\frac{g}{\gamma}}$ together with $g = \det(g_{ij})$ and $\gamma = \det(\gamma_{ij})$. Here the vertical bar $(\,|\,)$ stands for γ –covariant differentiation and T_i^j is the energy–momentum tensor of matter fields.

Several aspects of bimetric theory of gravitation have been studied by Rosen (1974, 1977), Goldman (1976), Karade (1980), Katore et al. (2006), Isrelit (1981), Khadekar et al. (2007). In particular, Reddy et al. (1998) have obtained some Bianchi Type cosmological models in bimetric theory of gravitation. The purpose of Rosen's bimetric theory is to get rid of the singularities that occur in general relativity that was appearing in the big–bang in cosmological models and therefore, recently, there has been a lot of interest in cosmological models in related to Rosen's bimetric theory of gravitation.

In the context of general relativity cosmic strings do not occur in Bianchi Type models. In it some Bianchi Type cosmological models – two in four and one in higher dimensions– are studied by Krori et al. (1994). They showed that the cosmic strings do not occur in Bianchi Type V cosmology. Bali and Dave (2003), Bali and Upadhaya (2003), Bali and Singh (2005) have investigated Bianchi Type IX, I and V string cosmological models under different physical conditions in general relativity. Raj Bali and Anjali (2006) have investigated Bianchi Type I magnetized string cosmological model in general relativity by introducing the condition $A=(BC)^n$, where $n > 0$ in Einstein field equations, whereas Raj Bali and Umesh Kumar Pareek (2007) have deduced Bianchi Type I string dust cosmological model with magnetic field in general relativity by imposing the condition $A= N\ (BC)^{-n}$, where $n > 0$ and N is proportionality constant, in Einstein field equations. Further Borkar et al. (2009, 2010), Gaikwad et al. (2011) and Kandalkar et al. (2011) have been investigated many magnetized cosmological models in bimetric theory of gravitation by using the techniques of Letelier and Stachel [(1979, 1983), 1980].

In this paper, some magnetized cosmological model with Λ -term in Rosen's bimetric theory of gravitation is investigated by using the techniques of Letelier and Stachel [(1979, 1983), 1980] and using the condition $A=(BC)^n$, where $n > 0$ in Rosen's field equations. The physical and geometrical significance of the model are discussed in presence and in absence of the magnetic field as well as in the presence and in the absence of cosmological constant Λ.

SOLUTIONS OF ROSEN'S FIELD EQUATIONS

We consider Bianchi Type I metric in the form

$$ds^2 = -\, d\,t^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2 \quad , \qquad (4)$$

where A, B and C are functions of t alone. Here $B \neq C$ otherwise we get LRS Bianchi Type I model.

The flat metric corresponding to metric (4) is

$$d\eta^2 = -\, dt^2 + dx^2 + dy^2 + dz^2 . \qquad (5)$$

The energy momentum tensor T_i^j for the string dust with magnetic field is taken as

$$T_i^j = (\in + p)\, \upsilon_i \upsilon^j + p\, g_i^j - \lambda\, x_i x^j + E_i^j \ , \qquad (6)$$

with

$$\upsilon^i\ \upsilon_i = -x_i\ x^i = -1, \qquad (7)$$

$$\upsilon^i x_i = 0 . \qquad (8)$$

In this model $\in$ and λ denote the rest energy density and the string tension density of the system respectively, p is the pressure, υ^i is the flow vector and x_i the direction of strings.

The electromagnetic field E_{ij} is given by Lichnerowicz (1967)

$$E_{ij} = \vec{\mu}\left[\,|\,h\,|^2\left(\upsilon_i\ \upsilon_j + \frac{1}{2} g_{ij}\right) - h_i\ h_j\right] . \qquad (9)$$

The four velocity vector υ_i is given by

$$g_{ij}\ \upsilon^i \upsilon^j = -1 . \qquad (10)$$

and $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijkl} F^{kl} \upsilon^j \ , \qquad (11)$$

where F_{kl} is the electromagnetic field tensor and $\in_{ijkl}$ is the Levi Civita tensor density.
Assume the comoving coordinates and hence we have

$\upsilon^1 = \upsilon^2 = \upsilon^3 = 0,\ \upsilon^4 = 1$.

Further we assume that the incident magnetic field is taken along *x*–axis, so that
$h_1 \neq 0, \quad h_2 = h_3 = h_4 = 0$.

The first set of Maxwell's equation

$$F_{[ij,k]} = 0 \ , \qquad (12)$$

yield
$F_{23} = \text{constant} = H(say)$.

Due to the assumption of infinite electrical conductivity, we have
$F_{14} = F_{24} = F_{34} = 0$.
The only non–vanishing component of F_{ij} is F_{23}.

So that

$$h_1 = \frac{AH}{\bar{\mu} BC} \tag{13}$$

and

$$|h|^2 = \frac{H^2}{\bar{\mu}^2 B^2 C^2} . \tag{14}$$

From equation (9), we obtain

$$-E_1^1 = E_2^2 = E_3^3 = -E_4^4 = \frac{H^2}{2\bar{\mu} B^2 C^2} . \tag{15}$$

Equation (6) of energy momentum tensor yield

$$T_1^1 = \left(p - \lambda - \frac{H^2}{2\bar{\mu} B^2 C^2}\right), T_2^2 = T_3^3 = \left(p + \frac{H^2}{2\bar{\mu} B^2 C^2}\right), T_4^4 = \left(-\in - \frac{H^2}{2\bar{\mu} B^2 C^2}\right) . \tag{16}$$

The Rosen's field equations (3) for the metric (4) and (5) with the help of (16) gives

$$\frac{-A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC\left(-p + \lambda + \frac{H^2}{2\bar{\mu} B^2 C^2}\right) - 2\Lambda , \tag{17}$$

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC\left(-p - \frac{H^2}{2\bar{\mu} B^2 C^2}\right) - 2\Lambda , \tag{18}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} = 16\pi ABC\left(-p - \frac{H^2}{2\bar{\mu} B^2 C^2}\right) - 2\Lambda , \tag{19}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4^2}{A^2} - \frac{B_4^2}{B^2} - \frac{C_4^2}{C^2} = 16\pi ABC\left(\in + \frac{H^2}{2\bar{\mu} B^2 C^2}\right) - 2\Lambda , \tag{20}$$

where $A_4 = \frac{dA}{dt}$, $B_4 = \frac{dB}{dt}$, $C_4 = \frac{dC}{dt}$ etc.

From equations (18) and (19), we obtain

$$\frac{C_{44}}{C} - \frac{B_{44}}{B} = \frac{C_4^2}{C^2} - \frac{B_4^2}{B^2} . \tag{21}$$

Equations (17) and (18) leads to

$$2\frac{B_{44}}{B} - 2\frac{A_{44}}{A} + 2\frac{A_4^2}{A^2} - 2\frac{B_4^2}{B^2} = 16\pi ABC\left(\lambda + \frac{2K}{B^2 C^2}\right) , \tag{22}$$

where

$K = \frac{H^2}{2\bar{\mu}}$.

Equations (20) and (22), after using string dust condition $(\in = \lambda)$ [Zel'dovich(1980)], lead to

$$\frac{B_{44}}{B} - 3\frac{A_{44}}{A} - \frac{C_{44}}{C} + 3\frac{A_4^{\ 2}}{A^2} - \frac{B_4^{\ 2}}{B^2} + \frac{C_4^{\ 2}}{C^2} = 16\pi A\left(\frac{K}{BC}\right) + 2\Lambda . \qquad (23)$$

Equations (17) to (20) are four equations, in six unknowns $A, B, C, \lambda, \in$ and p and therefore to deduce a determinate solution, we assume two extra conditions. First is that the component σ_1^1 of shear tensor σ_i^j is proportional to the expansion (θ) which leads to

$$A = (BC)^n \text{, where } n > 0 \qquad (24)$$

and second is Zel'dovich condition
$\in = \lambda$.

Using the first condition (equation (24)), in equation (23) , we write

$$(1-3n)\frac{B_{44}}{B} - (3n+1)\frac{C_{44}}{C} + (3n-1)\frac{B_4^{\ 2}}{B^2} + (3n+1)\frac{C_4^{\ 2}}{C^2} = 16\pi K(BC)^{n-1} + 2\Lambda . \quad (25)$$

Now the equation (21), rewrite as

$$\frac{(CB_4 - BC_4)_4}{(CB_4 - BC_4)} = \frac{B_4}{B} + \frac{C_4}{C} , \qquad (26)$$

which on integrating , we get

$$C^2\left(\frac{B}{C}\right)_4 = LBC . \qquad (27)$$

where L is the constant of integration.

Assume $BC = \mu,\ \frac{B}{C} = v$ then $B^2 = \mu v,\ C^2 = \frac{\mu}{v}$. In view of these relations, equation (27) becomes

$$\frac{v_4}{v} = L . \qquad (28)$$

Now equation (25) after using (24) and assumptions $BC = \mu$ and $\frac{B}{C} = v$ leads to

$$-3n\frac{\mu_{44}}{\mu} + 3n\left(\frac{\mu_4^2}{\mu^2}\right) - \left(\frac{v_4^{\ 2}}{v^2}\right) + \frac{v_{44}}{v} = 16\pi\mu^{(n-1)} K + 2\Lambda . \qquad (29)$$

The expressions (28) and (29) yield

$$2\mu_{44} - 2\frac{\mu_4^2}{\mu} = -\frac{4}{3n}\left[8\pi\mu^n K + \Lambda\,\mu\right], \tag{30}$$

which reduces to

$$\frac{d}{d\mu}\left[f^2\right] - 2\frac{f^2}{\mu} = -\frac{4}{3n}\left[8\pi\mu^n K + \Lambda\,\mu\right] , \tag{31}$$

where $\mu_4 = f(\mu)$.

The differential equation (31) has solution

$$f^2 = P\mu^2 - \frac{32\pi K\mu^{n+1}}{3n(n-1)} - \frac{4\Lambda}{3n}\mu^2 \log\mu , \tag{32}$$

where P is the constant of integration.

From equation (28) we write

$$\log v = \int \frac{L\, d\mu}{\sqrt{P\mu^2 - \frac{32\pi K\mu^{n+1}}{3n(n-1)} - \frac{4\Lambda}{3n}\mu^2 \log\mu}} + \log b , \tag{33}$$

where b is the constant of integration.

Now, using $\mu_4 = f(\mu)$ and expression (32), the metric (4) will be

$$ds^2 = -\frac{d\mu^2}{\left[P\mu^2 - \frac{32\pi K\mu^{n+1}}{3n(n-1)} - \frac{4\Lambda}{3n}\mu^2 \log\mu\right]} + \mu^{2n}dx^2 + \mu v\, dy^2 + \frac{\mu}{v}dz^2 , \tag{34}$$

where v is determined by equation (33).

After suitable transformation of co–ordinates $\mu = T,\ x = X,\ y = Y,\ z = Z$ i.e., $d\mu = dT,\ dx = dX,\ dy = dY,\ dz = dZ$, above metric can be re–written as

$$ds^2 = -\frac{dT^2}{\left[PT^2 - \frac{32\pi KT^{n+1}}{3n(n-1)} - \frac{4\Lambda}{3n}T^2 \log T\right]} + T^{2n}dX^2 + T v\; dY^2 + \frac{T}{v}dZ^2 . \tag{35}$$

Now choosing the cosmic time $u = \pm\log T$. For convenience, we can select $u = -\log T$, then the model (35) goes over to

$$ds^2 = -\frac{du^2}{\left[P - \frac{32\pi}{3n(n-1)}K(e^{-u})^{n-1} + \frac{4\Lambda}{3n}\; u\right]} + e^{-u}[(e^{-u})^{2n-1}dX^2 + v\; dY^2 + \frac{1}{v}dZ^2]. \tag{36}$$

This is the Bianchi Type I magnetized cosmological model with Λ - term in bimetric theory of gravitation.

PHYSICAL QUANTITIES

The energy density $(\in)$, the string tension density (λ), the pressure (p) for the model (36) (in terms of cosmic time u) are given by

$$\in (= \lambda) = \frac{(2n-1)\Lambda}{24 n \pi} (e^{u})^{n+1} - \frac{(4n+1)}{3n} K e^{2u} \ , \quad 0 < n \neq \frac{1}{2} \tag{37}$$

$$p = \frac{-\Lambda}{12\pi} (e^{u})^{n+1} - \frac{2}{3} K e^{2u} \ . \tag{38}$$

respectively. For $\Lambda \neq 0$ and finite u, the condition $\in > 0$ and $p > 0$ satisfies if

$$8\pi \frac{(4n+1)}{(2n-1)} K \ (e^{-u})^{n-1} < \Lambda < -8\pi K \ (e^{-u})^{n-1} \ . \tag{39}$$

and $\in = 0$, $p = 0$ gives $n = 0$, which contradicts to the fact that $n > 0$. Thus the model never goes to vacuum model, for finite value of u and for $\Lambda \neq 0$.

The strong energy conditions of Hawking and Ellis (1973): $(\in - \ p) > 0$ and $(\in + \ p) > 0$ are satisfies by our model (36) if

$$8\pi \ \frac{(2n+1)}{(4n-1)} K (e^{-u})^{n-1} < \Lambda < -8\pi \ (6n+1) \ K (e^{-u})^{n-1} . \tag{40}$$

The expansion θ given by $\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right)$ and for the model (36), it has the value

$\theta = (n+1) f \ e^{u}$,

which (after using (32)) reduces to

$$\theta = (n+1) \left[P - \frac{32\pi K (e^{-u})^{n-1}}{3n(n-1)} + \frac{4\Lambda u}{3n} \right]^{\frac{1}{2}} . \tag{41}$$

The components of shear tensor (σ_i^j) are given by

$$\sigma_1^1 = \frac{1}{3} \left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} \right]$$

or

$$\sigma_1^1 = \frac{(2n-1)}{3} \left[P - \frac{32\pi K (e^{-u})^{n-1}}{3n(n-1)} + \frac{4\Lambda u}{3n} \right]^{\frac{1}{2}} , \tag{42}$$

$$\sigma_2^2 = -\frac{1}{2} \ \sigma_1^1 + \frac{L}{2} \ , \tag{43}$$

$$\sigma_3^3 = -\frac{1}{2}\ \sigma_1^1 - \frac{L}{2}\ , \qquad (44)$$

$$\sigma_4^4 \ = \ 0\ . \qquad (45)$$

Special Case: In the absence of magnetic field *i.e.* for $K = 0$**,** we yield

$$\in\ (=\lambda) = \frac{(2n-1)\Lambda}{24\,n\,\pi}\ (e^u)^{n+1}\ . \qquad (46)$$

$$p = \frac{-\Lambda}{12\,\pi}(e^u)^{n+1}\ . \qquad (47)$$

$$\theta \ = \ (n+1)\left[P\ + \frac{4\Lambda\ u}{3n}\ \right]^{\frac{1}{2}}\ . \qquad (48)$$

$$\sigma_1^1 \ = \ \frac{(2n-1)}{3}\left[P\ + \frac{4\Lambda\ u}{3n}\ \right]^{\frac{1}{2}}\ , \qquad (49)$$

$$\sigma_2^2 \ =- \ \frac{(2n-1)}{6}\left[P\ + \frac{4\Lambda\ u}{3n}\ \right]^{\frac{1}{2}} + \frac{L}{2}\ , \qquad (50)$$

$$\sigma_3^3 \ =- \frac{(2n-1)}{6}\left[P\ + \frac{4\Lambda\ u}{3n}\ \right]^{\frac{1}{2}} - \frac{L}{2}\ , \qquad (51)$$

$$\sigma_4^4 \ = 0\ \ . \qquad (52)$$

CONCLUSION

There is no big-bang and big crunch singularities in our model (36). It is interesting to note that the cosmological constant Λ playing the important role in our model. From the equations (37), (38) and (39), it is seen that the rest energy density $\in$, string tension density λ and pressure p all are positive if Λ lies in the open interval, equation (39), for n is not equal to half and for finite u. This shows that such a Bianchi Type I magnetized cosmological model (36) in bimetric theory of gravitation exists when cosmological constant Λ lies in the open interval equation (39). It never goes to vacuum model for Λ is not equal to zero and for finite value of u. When $u \to -\infty$ then $\theta \to -\infty$, $\sigma \to -\infty$ and when $u \to \infty$ then $\theta \to \infty$, $\sigma \to \infty$. This confirms that the model is expanding as well as shearing. The expansion and the shear in the model increases as cosmic time u increases. Further when $u \to \infty$ then $\in = \lambda \to \infty$, $p \to \infty$. When $u \to -\infty$ then $\in = \lambda \to 0$ and $p \to 0$, which shows that our model goes over to vacuum model, when the cosmic time u is minus infinity.

In the special case, in the absence of magnetic field i.e., $K = 0$, from the equation (46) and (47), it is seen that our model (36) exists if Λ is negative and n between zero and half and if the cosmological constant $\Lambda > 0$, then our model filled with dark matter for n greater than half, for finite value of cosmic time u in the absence of magnetic field K. Also the expansion and shear in the model increases, as the cosmic time u increases.

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References

1. Bali R. and Anjali. (2006). Bianchi Type I Magnetized String Cosmological Model in General Relativity, Astrophysics Space Science, Vol. 302.
2. Bali R. and Dave S. (2003). Bianchi Type -IX String Cosmological Models with Bulk Viscous Fluid in General Relativity, Astrophysics Space Science, Vol. 288.
3. Bali R . and Upadhaya R. D. (2003). LRS Bianchi Type I String Dust Magnetized Cosmological Models, Astrophysics Space Science, Vol. 283.
4. Bali R. and Singh D. (2005). Bianchi Type–V Bulk Viscous Fluid String Dust Cosmological Model in General Relativity, Astrophysics Space Science, Vol. 300.
5. Bali R. and Pareek U. K. (2007). Bianchi Type I String Dust Cosmological Model with Magnetic Field in General Relativity, Astrophysics Space Science, Vol. 312.
6. Borkar M. S. and Charjan S. S. (2009). Bianchi Type I String Dust Cosmological Model with Magnetic Field in Bimetric Relativity, International Journal of Applied Mathematics, Vol. 22, No. 3.
7. Borkar M. S. and Charjan S. S. (2010). The Charged Perfect Fluid Distribution in Bimetric Theory of Relativity, Journal of the Indian Academy Mathematics, Vol. 32, No. 1.
8. Borkar M. S. and Charjan S. S. (2010). Bianchi Type I Bulk Viscous Fluid String Dust Cosmological Model with Magnetic Field in Bimetric Theory of Gravitation, International Journal Applications and Applied Mathematics, Vol. 5, No. 1.
9. Borkar M. S. and Charjan S. S. (2010). Bianchi Type I Magnetized Dust Cosmological Model in Bimetric Theory of Gravitation, International Journal of Applications and Applied Mathematics, Vol. 5, No. 2.
10. Gaikwad N. P., Borkar M. S. and Charjan S. S. (2011). Bianchi Type I Massive String Magnetized Baratropic Perfect Fluid Cosmological Model in Bimetric Theory of Gravitation, Chinese Physics Letter, Vol. 28, No. 8.
11. Borkar M. S. and Charjan S. S. (2013),LRS Bianchi Type I Magnetized Cosmological Model with Λ --Term in Bimetric Theory of Gravitation, American Journal of Pure Applied Mathematics, 2(1), pp.7-12
12. Borkar M. S. and Charjan S. S, V.V. Lepse *(2014),* LRS Bianchi Type I Cosmological Models with Perfect Fluid and Dark Energy in Bimetric Theory of Gravitation, *IOSR Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728, p-ISSN:2319-765X.* 10(3) *Ver. I PP 62-69*
13. Borkar M. S. and Charjan S. S and Lepse V.V. (2016),, Bianchi Type-I Hyperbolic Models with Perfect Fluid and Dark Energy in Bimetric Theory of Gravitation, *An Int. J. AAM* 11 (1), , pp. 192 - 214
14. Charjan S.S.,. Dhongade P. R , Borkar M.S. (2019) , Bianchi Type I Bulk Viscous Fluid String Dust Magnetized Cosmological Model with Λ -Term in Bimetric Theory of Gravitation, International journal of Scientific Research in Mathematical and statistical sciences, 6 (3) pp.35-40, E-ISSN: 2348-4519 , *DOI: https://doi.org/10.26438/ijsrmss/v6i3.3540 ,*
15. Goldman I. and Rosen N. (1976). A Cosmological Model in the Bimetric Gravitation Theory, The Astrophysical Journal, Vol. 212.

16. Hawking S.W. and Ellis G.F.R. (1973). The Large Scale Structure of Space-Time, Cambridge University Press, Cambridge.
17. Isrelit M. (1981). Spherically Symmetric Fields in Rosen's Bimetric Theories of Gravitation, General Relativity and Gravitation, Vol. 13, No. 7.
18. Kandalkar S. P. et al. (2011). String Cosmology in Kantowski Sachs Space-Time with Bulk Viscosity and Magnetic Field, Journal of Vectorial Relativity, Vol. 6.
19. Karade T. M. (1980). Spherically Symmetric Space Times in Bimetric Relativity Theory I, Indian Journal of Pure-Applied Mathematics, Vol. 11, No. 9.
20. Katore S.D. and Rane R.S. (2006). Magnetized Cosmological Models in Bimetric Theory of Gravitation, Pramana Journal of Physics, Vol. 67, No. 2.
21. Khadekar G.S. and Tade S.D. (2007). String Cosmological Models in Five Dimensional Bimetric Theory of Gravitation, Astrophysics Space Science, DOI 10.1007\S10509-007-9410-2.T.
22. Krori K.D., Choudhuri T. and Mahant C.R. (1994). String in some Bianchi Type Cosmologies, General Relativity and Gravitation, Vol. 26, No. 3.
23. Letelier P.S. (1979). Clouds of Strings in General Relativity, Physical Review D, Vol. 20, No. 6.
24. Letelier P.S. (1983). String Cosmology, Physical Review D, Vol. 28, No. 10.
25. Lichnerowicz A. (1967). Relativistic Hydrodynamics and Magneto hydrodynamics, Benjamin, New York.
26. Reddy D.R.K. and Venkateshwara Rao N. (1998). On Some Bianchi Type Cosmological Models in Bimetric Theory of Gravitation, Astrophysics Space Science, Vol. 257.
27. Rosen N. (1974). A Theory of Gravitation, Annls of Physics, Vol. 84.
28. Rosen N. (1977). Topics in Theoretical and Experiential Gravitation Physics, Edited by Sabbata V. De and Weber J., Plenum Press, London.
29. Stachel J. (1980). Thickening the String I, The String Perfect Dust, Physical Review D, Vol. 21, No. 8.
30. Zel'dovich Y.B. (1980). Cosmological Fluctuations Produced Near a Singularity, Monthly Notices of the Royal Astronomical Society, Vol. 192.